Problem 1 (8 Punkte)
Let X_n, X be compact metric spaces. Prove the following statements.

(a) $(X_n, o_n) \overset{GH}{\to} (X, o)$ implies $X_n \overset{GH}{\to} X$.

(b) If $X_n \overset{GH}{\to} X$ and $o \in X$, then one can choose $o_n \in X_n$ such that $(X_n, o_n) \overset{GH}{\to} (X, o)$.

(c) Consider pointed metric spaces $(X_n, o_n), n \in \mathbb{N}$ and (X, o) such that $(X_n, o_n) \overset{GH}{\to} (X, o)$. Assume X is a length space. Then $B_r(o_n) \overset{GH}{\to} B_r(o)$ for all $r > 0$.

(d) Let $(X_n, o_n) \overset{GH}{\to} (X, o)$. Asume that X_n is boundedly compact and X is complete. Then X is boundedly compact.

Problem 2 (4 Punkte)
Prove that the quadruple condition is equivalent to the following modified quadrupel condition: For every $x \in X$ there exists an open neighborhood U such that for any quadruple $(a; b, c, d)$ in U there exists a quadruple $(\bar{a}; \bar{b}, \bar{c}, \bar{d})$ in the k-plane such that the segments $[\bar{a}, \bar{b}]$, $[\bar{a}, \bar{c}]$ and $[\bar{a}, \bar{d}]$ devide the full angle at $\bar{a}$ into 3 angles less than π where $|\bar{a}\bar{b}| = |ab|$, $|\bar{a}\bar{c}| = |ac|$, $|\bar{a}\bar{d}| = |ad|$, and $|\bar{b}\bar{c}| \geq |bc|$, $|\bar{c}\bar{d}| \geq |cd|$ and $|\bar{d}\bar{b}| \geq |db|$.

Bonus Problem (4 Punkte)
Construct a metric space X with two points $p, q \in X$ such that the balls $B_r(p)$ and $B_r(q)$ are isometric for every $r > 0$, but there is no isometry from X to itself that sends p to q.

Hint: There is an example among finite spaces.

Abgabe am Donnerstag, 2. Februar bis 12 Uhr beim Assistenten.