Abteilung für mathematische Logik
Mathematische Logik (SS 2014)
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Übung zur Vorlesung
Hinweis, Aufgabe 6, Überprüfung
Abgabe am 16.6 vor 16:00 Uhr

Aufgabe (6-1-a)**.** An algebraically closed field is a field in which every polynomial of every degree has a root. That every polynomial of every degree has a root cannot be written as a single sentence. Be careful not to write this:

$$\forall n \forall a_1, \ldots, a_n \exists x (a_0 + \ldots + a_n x^n = 0),$$

because as you know by now, quantifying over natural numbers is not allowed. Characteristic zero means that for every n, $n.1$ is not zero. By $n.1$ I mean $\underbrace{1 + 1 + \ldots + 1}_{n \text{ times}}$.

Aufgabe (6-1-b)**.** Let K be the class of algebraically closed fields of characteristic zero, axiomatised by Σ. In Aufgabe 5-4 you proved that if this theory is finitely axiomatisable, then there is a theory that axiomatises the complement of K. Call this Σ^c. Check that $\Sigma^c \cup \{n.1 \neq 0, n \in \mathbb{N}\}$ is consistent (check that every finite subset of it, is).

Aufgabe (6-1-c)**.** You should refer to a theorem in the script that guarantees this. This theorem is one of the corollaries of compactness theorem.

Aufgabe (6-1-d)**.** Check that every 'quantifier-free' formula (with parameters) in the language of rings is a conjunction ($\vee$) of formulas of the form

$$\bigwedge_{i=1}^{n} P_i(x) = 0 \wedge \bigwedge_{i=1}^{m} Q_i(x) \neq 0$$

where P_i and Q_i are polynomials (with coefficients in the structure, here M). Consider two cases: case 1, when there is a non-zero polynomial among P_i's. Note that your fields are algebraically closed. Case 2, where all P_i are zero and your formula is of the form: $\bigwedge_{i=1}^{m} Q_i(x) \neq 0$.

Aufgabe (6-1-e)**.** Let $\mathfrak{N} \models \phi(\bar{a})$ with $\bar{a} \in M$. Then $\mathfrak{N} \models \psi_\phi(\bar{a})$ for some quantor-free ψ_ϕ. We also have the following: if M is a substructure of N and ψ is a quantifier free formula with parameters in M, then $M \models \psi$ if and only if $N \models \psi$.

Aufgabe (6-1-g)**.** Let $\phi \in \mathrm{Th}(\mathfrak{C})$. Then $\Sigma \models \phi$ or $\Sigma \models \neg\phi$. Continue.