**Universität Freiburg, Abteilung für Mathematische Logik**
Übung zur Vorlesung Modelltheorie 2, ss2015
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## Blatt 11, prime Erweiterungen

**Aufgabe 1.** Let $T$ be totally transcendental. Show that the prime extensions are unique.

**Aufgabe 2.**

1. Let $M = (b_\alpha)$ be a construction over $A$ and $C \subseteq M$ be construction closed. Show that for each $b_\alpha \in C$,

$$\mathrm{tp}(b_\alpha/A(b_{<\alpha} \cap C)) \vdash \mathrm{tp}(b_\alpha/AB_\alpha).$$

   with the definition in the next item, this means that $b_\alpha$ and $Ab_{<\alpha}$ are weakly orthogonal over $A(b_{<\alpha} \cap C)$:

$$b_\alpha \underset{A(b_{<\alpha}\cap C)}{\overset{w}{\perp}} Ab_{<\alpha}$$

2. Two types $p(x)$ and $q(y)$, both in $S(A)$, are called **weakly orthogonal** if $p(x) \cup q(y)$ determines a complete type in $x, y$. Show that $p$ and $q$ are weakly orthogonal if for every $a \models p$, the type $q$ has a unique extension to $aA$.

**Aufgabe 3.** Let $T$ be countable. Show that the following are equivalent:

1. Every parameter set has a prime extension.

2. Over every countable parameter set, the isolated types are dense.

3. Over every parameter set the isolated types are dense.

**Aufgabe 4.** Show that the following two versions of Fodor's theorem are equivalent.

1. If $\{C_\alpha\}_{\alpha<\omega_1}$ are clubs, then

$$C := \{\alpha | \alpha \in \bigcap_{\beta<\alpha} C_\beta\} \text{ (the diagonal intersection of the } C_\alpha\text{)}$$

   is also a club.

2. If $D$ is a club and $f : D \to \omega_1$ is regressive, then there is an ordinal $\beta$ such that $\{\alpha \in D | f(\alpha) = \beta\}$ is stationary.

**Aufgabe 5.** What do you think is a better option for the next Monday?

1. Solving random exercises.
2. Beginning with Hrushovski's constructions.
3. Neither!