Universität Freiburg, Abteilung für Mathematische Logik
Übung zur Vorlesung Modelltheorie 2, ss2015
Prof. Dr. Martin Ziegler
Dr. Mohsen Khani

Blatt 7, Stabilität, Erbe=Coerbe=eindeutige nichtforkende Erweiterung

In this frame I have provided all material needed for solving the exercises in this sheet.

Bemerkung.

1. Let $M \subseteq B$ and M be a model. If $p \in S(M)$ is definable, then it has a unique heir $q \in S(B)$. The type q is also definable over M and
$$q = \{\phi(x, \bar{b}) | \phi(x, \bar{y}) \in L, \bar{b} \in B, \mathfrak{C} \models d_p x \phi(x, \bar{b})\}.$$
2. If T is stable and $p \in S(A)$ is a type, then p is definable over A.
3. In stable theories, every type $p \in S(M)$ has a unique heir $q \in S(B)$ (for B a paremeter set extending the model M). This unique heir, is also the unique coheir, and the unique non-forking extension of p.
4. Stable theories are simple, so forking = dividing.
5. If $A \subseteq B$ and π is a partial type over B not forking over A, then π extends to a complete type over B not forking over A.

Bemerkung (Harrington)**.** Let T be a stable theory and p and q be global types. For every formula $\phi(x, y)$ without parameters

$$d_p x \phi(x, y) \in q(y) \Leftrightarrow d_q y \phi(x, y) \in p(x).$$

Aufgabe 1. Let I be an indiscernible sequence over a parameter set A. Show that there exists a models M containing A such that I is indiscernible over M.

Use the above Aufgabe, to prove the following:

Aufgabe 2. Let A be a parameter set. if $\phi(x, b)$ is satisfiable in every model M containing A, then $\phi(x, b)$ does not divide over A (in stable theories, the converse holds too; the next Aufgabe).

Aufgabe 3. Suppose that T is a stable theory. A formula $\phi(x, b)$ does not fork over a parameter set A if and only if $\phi(x, b)$ is satisfiable in every model M containing A (use item 3 and 5 in the frame and the previous Aufgabe).

Aufgabe 4. We call q a **weak heir** of p if it satisfies the definition of heir for formulae without parameters. Precisely, let $M \models T$, $M \subseteq B$, $p \in S(M)$ and $q \in S(A)$. Call q a weak heir of p if for every formula $\phi(x, y)$ **without parameters** and each $b \in B$,

$$\phi(x, b) \in q \Rightarrow \exists m \in M \quad \phi(x, m) \in p.$$

Show that in stable theories, weak heir and heir are the same.

Aufgabe 5. Assumptions: T is stable, M is a model, A is a parameter set, $M \subseteq A$, $p \in S(M)$ and $q \in S(A)$. Use the theorem of Harrington stated in the frame to prove that the following are equivalent:

1. q is an (the) heir of p.
2. q is a (the) coheir of p.