Universitt Freiburg, Abteilung für Mathematische Logik
Übung zur Vorlesung Modelltheorie 1, ws2014-2015
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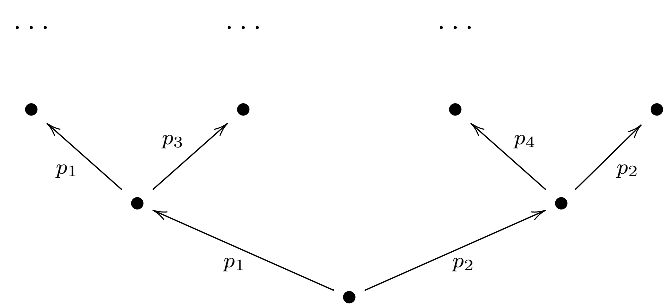
Blatt 10, Prime Models and Indiscernible Sequences

Solving 3 exercises suffices. You may always add to your points by solving more. The tutors are not required to provide detailed solutions for more than 4 exercises.

Aufgabe 1 (number of types and binary trees)**.** Suppose that T is a countable theory in which there is no binary tree of consistent formulae. Show that for each n, $|S_n(T)|$ is at most countable (the converse also holds and is a theorem in the script: if T is such that for each n, $|S_n(T)|$ is at most countable, then there is no binary tree of consistent formulae in T).



Aufgabe 2. Show that for any infinite L-structure M, we can find

$$N_0 \succ N_1 \succ N_2 \succ N_3 \succ \dots,$$

a descending *elementary* chain of *elementary* extensions of M, such that $M = \bigcap_{i \in \mathbb{N}} N_i$.

Hinweis. Clues: L(Skolem), an indiscernible sequence $(a_i)_{i\in\mathbb{N}}$, N_0 being obtained from M and $(a_i)_{i\in\mathbb{N}}$.

Aufgabe 3.

1. Show that ACF (the theory of algebraically closed fields) has a prime model.
2. Show that RCF (the theory of real closed fields) has a prime model.

3. Show that $\mathrm{Th}(\mathbb{N})$ in the language $L = \{+, \cdot, <, 0, 1\}$ has a prime model.

4. Let T be the theory of $(\mathbb{R}, <, Q)$ where Q is a predicate for rational numbers. Does T have a prime model?

Aufgabe 4. Let (G, R) be an infinite graph. Use Ramsey's theorem to show that either G has an infinite complete subgraph (a subgraph in which there is an edge between any two vertices) or it has infinite null subgraph (=there are infinitely many vertices in G with no edges in between).

Aufgabe 5. Show that if M is κ-saturated, then there is $I \subseteq M$, a sequence of order indiscernibles with $|I| = \kappa$.

Aufgabe 6. Suppose that $K \models$ACF and K has infinite transcendence degree. Let $I = \{a_1, a_2, \ldots, \}$ be an infinite algebraically independent set (its elements are algebraically independent over $\mathbb{Q}$). Show that I is an infinite set of indiscernibles in K.

Aufgabe 7. Show that there is no $\aleph_0$-categorical theory of fields. That is if T is a complete theory in the language of rings that contains the theory of fields, then T is not $\aleph_0$-categorical.

Hinweis. We have proved that if T is $\aleph_0$-categorical then the algebraic closure of a finite set is finite (Blatt 9 Auf 5)