Universität Freiburg, Abteilung für Mathematische Logik
Übung zur Vorlesung Modelltheorie 1, ws2014-2015
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Blatt 12, Vaughtian Pairs, Prime Extensions, Indiscernibles

Aufgabe 1. Show that a sequence of elements in $(\mathbb{Q}, <)$ is indiscernible if and only if it is either constant, strictly increasing or strictly decreasing.

Aufgabe 2. Show that for a countable T the following are equivalent (show only $1 \to 2 \to 3$):

1. every parameter set has a prime extension;
2. the isolated types over any countable parameter set are dense;
3. the isolated types over any parameter set are dense.

Hinweis (Hinweise $1 \to 2$). Let A be a countable parameter set and M its prime extension and ϕ a formula with parameters in A. We want to show that $[\phi]$ (open set in the space of types) contains an isolated type. In other words we want ϕ to belong to an isolated type. There is an element $a \in M$ such that $M \models \phi(a)$. Show that $\mathrm{tp}(a/A)$ is isolated (use the omitting type theorem).

Aufgabe 3. Solve only one item below (they are both solved with the same idea).

1. Suppose that T is countable and complete and with infinite models. Suppose that $M \models T$ and $\phi \in L(M)$ and $\phi(M)$ is infinite with smaller cardinality than the cardinality of M. Show that there is an elementary substructure N of M ($N \prec M$) such that (M, N) is Vaughtian pair for ϕ and the cardinality of N equals to the cardinality of $\phi(M)$ (the converse also holds and is a theorem in the script: if T has a Vaughtian pair, then it has a model M with cardinality $\aleph_1$ and there is a $\phi \in L(M)$ such that $\phi(M)$ is countable).
2. If T (as above) has a Vaughtian pair, then it has a Vaughtian pair (M, N) in which M is countable.

Aufgabe 4. Solve only two items.

1. Show that the theory of the random graph has a Vaughtian pair.
2. Let $L = \{E\}$ be the language with a single binary relation symbol. Let T be the theory of an equivalence relation where for each $n \in \omega$ there is a unique equivalence class of size n. Exhibit a Vaughthian pair of models of T (remember that in Blatt 11 you have proved that T is ω-stable and not $\aleph_1$-categorical).
3. Show that there is no Vaughtian pair of real closed fields.