Universität Freiburg, Abteilung für Mathematische Logik
Übung zur Vorlesung Modelltheorie 1, ws2014-2015
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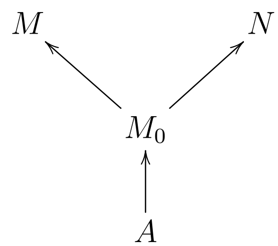
Blatt 14, Strong Minimality

Aufgabe 1. Consider the following diagram:



$$M_0 \prec M, M_0 \prec N$$
$$A \text{ subset of } M_0$$

1. let $\bar{a}$ be a tuple in A and $\phi(x, \bar{a})$ a formula. Then show that the fact that

   $$\phi(x, \bar{a}) \text{ defines a strongly minimal set in } M$$

   is an elementary property of $\bar{a}$ contained in the $\mathrm{tp}^M(\bar{a})$. It means that in the above diagram if $\phi(x, \bar{a})$ defines a strongly minimal set in M then it defines a strongly minimal set in N too.

2. Suppose that $a_1, \ldots, a_n$ in $\phi(M)$ are independent over A and $b_1, \ldots b_n \in \phi(N)$ are independent over A. Then show that $\mathrm{tp}^M(\bar{a}/A) = \mathrm{tp}^N(\bar{b}/A)$.

3. Let $B \subseteq \phi(M)$ be infinite and independent over A. Show that B is a set of indiscernibles over A (note that being a set of indiscernibles is a stronger property than being a sequence of indiscernibles).

4. Let $C \subseteq \phi(N)$ be infinite and independent over A. Show that C is a set of indiscernibles over A with the same type as type of B.

Aufgabe 2. 1. Let T be ω-stable. Show that if $M \models T$ then there is a minimal formula in M.

2. If $M \models T$ is $\aleph_0$-saturated and $\phi(\bar{x}, \bar{a})$ is minimal in M, then it is strongly minimal.

Aufgabe 3. 1. Show that the theory of K-vector spaces is κ-categorical for all $\kappa > |K|$.

2. Is ACF_p $\aleph_0$-categorical?