Universitt Freiburg, Abteilung für Mathematische Logik
Übung zur Vorlesung Modelltheorie 1, ws2014-2015
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Blatt 7, $\aleph_0$-categoricity and ω-saturatedness

Nur nummerierte Aufgaben sind abzugeben.

Aufgabe 1. Show that T is $\aleph_0$-categorical if and only if $S_n(T)$ is finite for each n.

Aufgabe 2. Suppose that M is countable and it is a model of an $\aleph_0$-categorical theory. Show that if $X \subseteq M^n$ is invariant under all automorphisms of M, then X is definable (compare with Blatt 1, Definierbarkeit).

Aufgabe. Aufgabe 2 in above can be generalised: Let M be saturated and A be a subset of M with $|A| < |M|$. Let $X \subseteq M^n$ be definable with parameters in M. Then X is A-definable if and only if every automorphism of M that fixes A pointwise, fixes X setwise (the only if part of the statement does not require that M is saturated).

Aufgabe 3. Axiomatise a theory with exactly two countable models (also remind yourself of Vaught's theorem that there is no countable complete theory with exactly two countable models).

Aufgabe 4. Suppose that M is ω-saturated. Show that N is partially isomorphic to M if and only if N is ω-saturated and elementarily equivalent to M (see Aufgabe 1 Blatt 3).

Aufgabe (a test for quantifier elimination)**.** Suppose that L is a language with at least one constant symbol and T is an L-theory. T has quantifier elimination if and only if whenever $M, N \models T$ and A is a subset of M and $f : A \to N$ is a partial embedding, f extends to an embedding of M into N.

$$\begin{tikzcd}
M \arrow[rr, dashed, "f \text{ extends}"] & & N \\
& A \arrow[ul, "\subseteq"] \arrow[ur, "f \text{ (partial embedding)}"'] &
\end{tikzcd}$$

$M, N \models T$

N saturated

A subset of M

$f : A \to N$ partial embedding