

Blatt 7, \aleph_0 -categoricity and ω -saturatedness

Nur nummerierte Aufgaben sind abzugeben.

Aufgabe 1. Show that T is \aleph_0 -categorical if and only if $S_n(T)$ is finite for each n .

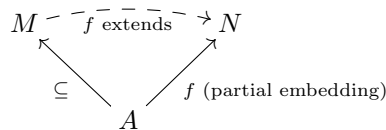
Aufgabe 2. Suppose that M is countable and it is a model of an \aleph_0 -categorical theory. Show that if $X \subseteq M^n$ is invariant under all automorphisms of M , then X is definable (compare with Blatt 1, Definierbarkeit).

Aufgabe. Aufgabe 2 in above can be generalised: Let M be saturated and A be a subset of M with $|A| < |M|$. Let $X \subseteq M^n$ be definable with parameters in M . Then X is A -definable if and only if every automorphism of M that fixes A pointwise, fixes X setwise (the only if part of the statement does not require that M is saturated).

Aufgabe 3. Axiomatise a theory with exactly two countable models (also remind yourself of Vaught's theorem that there is no countable complete theory with exactly two countable models).

Aufgabe 4. Suppose that M is ω -saturated. Show that N is partially isomorphic to M if and only if N is ω -saturated and elementarily equivalent to M (see Aufgabe 1 Blatt 3).

Aufgabe (a test for quantifier elimination). Suppose that L is a language with at least one constant symbol and T is an L -theory. T has quantifier elimination if and only if whenever $M, N \models T$ and A is a subset of M and $f : A \rightarrow N$ is a partial embedding, f extends to an embedding of M into N .



$M, N \models T$

N saturated

A subset of M

$f : A \rightarrow N$ partial embedding