Universitt Freiburg, Abteilung für Mathematische Logik
Übung zur Vorlesung Modelltheorie 1, ws2014-2015
Prof. Dr. Heike Mildenberger
Dr. Mohsen Khani

Blatt 8, complementary exercises for chapter 3

This sheet is intended for those who have interest in more involved algebraic exercises. While there is no need to hand in solutions, we will consider bonus points for each exercise you solve!

Aufgabe 1. Suppose that K is an algebraically closed field and $P \subseteq K[X_1, \ldots, X_n]$ is a maximal ideal. Show that P is generated by $X_1 - a_1, \ldots, X_n - a_n$ for some $a_1, \ldots, a_n \in K$.

Aufgabe 2 (completeness of projective varieties)**.** Let K be a model of ACF. Suppose that $p_1, \ldots, p_k \in \mathbb{Z}[\bar{Y}, \bar{X}]$ are homogeneous in $\bar{X}$ (i.e. $p_i(\bar{Y}, t\bar{X}) = t^d p_i(\bar{Y}, \bar{X})$ for some d). Let $\phi(\bar{y})$ be the formula that says that the system of equations $p_1(\bar{x}, \bar{y}) = \ldots = p_k(\bar{x}, \bar{y}) = 0$ has a nontrivial solution.

1. $\phi(\bar{y})$ is equivalent to a positive quantifier free-formula.
2. Let $\mathbb{P}^l$ be the projective l-space over K, and let $\pi : \mathbb{P}^n \times \mathbb{P}^m \to \mathbb{P}^m$ be the natural projection map. Show that π is a closed map in the Zariski topology.

Aufgabe 3. Let $K \subseteq L$ be algebraically closed fields. Let $V, W \subseteq L^n$ be Zariski closed sets defined over K. Suppose that there is $f : V \to W$ a bijective polynomial map defined over L. Show that there is $g : V \cap K^n \to W \cap K^n$ a bijective polynomial map defined over K.