Abteilung für mathematische Logik
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A Quick Remark

Sneaking through your "Abgaben", I noticed a very good misunderstanding that I need to settle here. You were supposed to write a formula with a certain interpretation in a language with only one relation symbol R. Many had written things like $R^{\mathfrak{M}}(x, y)$. Note carefully that $R^{\mathfrak{M}}(x, y)$ is "not" a formula in the language L.

What is the difference between $R(c_1, c_2)$ and $R^{\mathfrak{M}}(c_1^{\mathfrak{M}}, c_2^{\mathfrak{M}})$? In the first order logic (and other logics) we deal with two different worlds. The world of syntax and the world of semantic. The world of syntax is where we write formulae, and (let's say) we do not regard them as to have any "meaning". In the world of semantic, we take a formula from the syntax and then "attach a meaning" to it. Consider the Deutsch language. "ein Buch" is a sequence of symbols in this language, which by itself has no meaning. But when I say this word, in your mind you imagine a "book" and interpret this word as the picture you have imagined. So there is a function in your mind that is constantly sending a meaningless sequence of symbols to an object (meaning). Of course, if there is another world, in which people attach a different thing, say a "cow" to "ein Buch", then they will interpret whatever you say about "ein Buch" in a rather different way! So in the world of semantic, we need to know in which universe we are and what the interpretations of our symbols are in that universe. Back to our mathematics, $R(c_1, c_2)$ is a first order formula which does not have any meaning. Now, let $\mathfrak{M} = \langle M, ...\rangle$ be a suitable structure. In its universe M you attach the meaning $R^{\mathfrak{M}}(c_1^{\mathfrak{M}}, c_2^{\mathfrak{M}})$ to the formula $R(c_1, c_2)$. It means that you first consider what the meaning of c_1 in M is (which is $c_1^{\mathfrak{M}}$), and then the same for c_2, and then you check what R is interpreted of (i.e. $R^{\mathfrak{M}}$). Of course when the structure is $\mathfrak{N}$ you have other interpretations, $c_1^{\mathfrak{N}}$, $c_1^{\mathfrak{N}}$ and $R^{\mathfrak{N}}$.

I think you now know better what the following means:

$$M \models R(c_1, c_2) \text{ if } (c_1^{\mathfrak{M}}, c_2^{\mathfrak{M}}) \in R^{\mathfrak{M}}$$

The things after "if" are "not" first order. They are written in our daily mathematics language. Also as a simple exercise, determine what the difference between $\rightarrow$ and $\Rightarrow$ is.