Universität Freiburg, Abteilung für Mathematische Logik
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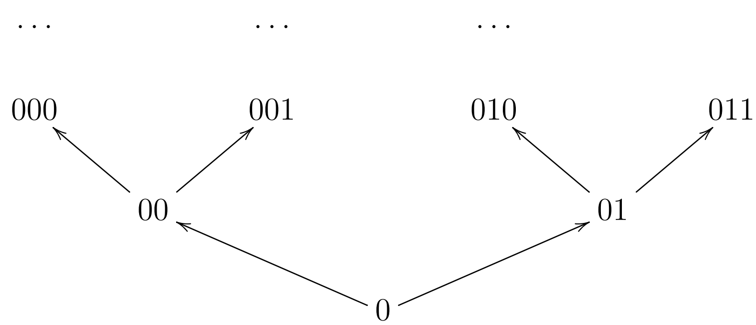
1 Settling ambiguities

Here is a short explanation on two topics that seem to have baffled some.

How many nodes and how many branches are there in a binary tree In a theorem in the script we were supposed to prove that theories with binary trees are not ω-stable. We needed a countable A for which $S(A)$ is uncountable. We simply said that every node of the tree gives a parameter in A, so there are countably many parameters; and every branch gives rise to a type, and hence there are uncountably many types.



The numer of nodes=the number of finite sequences of $0, 1$

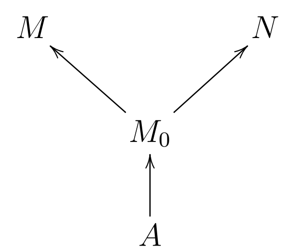
The number of branches=the numbe of countable sequence of $0, 1$

Note that whenever T is an infinite binary tree then the number of nodes is countable and the number of branches in uncountable. This is not too easy to imagine! (because we expect the number of nodes to be more than the number of branches, as it is the case in finite trees). But note that x is a node if x is represented by a finite sequence of 0's and 1's and f is a branch if it is an infinite sequence of 0's and 1' s.
The above diagram also clarifies the symbols $s^\frown 0$ and $s^\frown 1$.

What is a complete type? To this point, we have seen several definitions for a complete type and different notations corresponding to each: $\mathrm{tp}^M(a/A)$, $\mathrm{tp}(a/A)$, p, etc. If you are confused whether or not the superscript M must be there then read the following. What I want to explain is that the concept of types is an **elementary** concept.

So let us first review the definitions: we said that $p(x)$ is a type in $S^M(A)$ if $p(x)$ is a maximal set of formulae consistent with $\mathrm{Th}(M_A)$. Now since $p(x) \cup \mathrm{Th}(M_A)$ is consistent, (one can prove that) there exists N an elementary extension of M and an $a \in N$ such that $p(x) = \{\phi(x) \in L_A | N \models \phi(a)\}$. So we could have said that $p \in S^M(A)$ if there exists an extension N of M and an element $a \in N$ such that $p(x) = \mathrm{tp}^N(a)$. So $\mathrm{tp}^N(a/A)$ is a type in $S^M(A)$ and this seems to be the confusing part (if the superscript is N then why is it a type in S^M?). If a happens to be in M then of course $\mathrm{tp}^N(a/A) = \mathrm{tp}^M(a/A)$, but there is no guarantee that a must be in M. There could even be some other element b in M such that $\mathrm{tp}^N(a/A) = \mathrm{tp}^M(b/A)$.

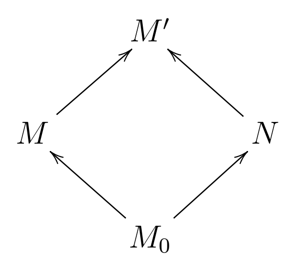
Consider the following diagram:



$M_0 \preceq M, N$ and A a subset of M_0

suppose that $a \in M$. Then as we mentioned above $\mathrm{tp}^M(a/A)$ is both in $S^M(A)$ and $S^{M_0}(A)$. It is (perhaps confusingly) also in $S^N(A)$. Because $p = \mathrm{tp}^M(A)$ is consistent with $\mathrm{Th}(M_A)$ and according to the diagram $\mathrm{Th}(M_A) = \mathrm{Th}(N_A) = \mathrm{Th}(M_{0_A})$. So $S^M(A) = S^N(A) = S^{M_0}(A)$.

We proved in an exercise that indeed on can find a structure M' as in the following diagram:



$$M_0 \preceq M, N \preceq M'$$

so one could have said that it is indeed $\mathrm{tp}^{M'}(a/A)$ that we are talking about, which is in $S^{M_0}(A)$ and $S^M(A)$ and $S^N(A)$. So perhaps it is a good idea to remove the name of the structure and say $\mathrm{tp}(a/A)$ where we mean in some M containing A and also in all elementary extensions and elementary substructures of it containing A such as M_0, N, M' in the diagram above. Considering this, we can also write $S(A)$ instead of $S^M(A)$, but we keep the diagram of elementary extensions in mind.

We often fix a model $\mathbb{M}$ which is big and saturated enough (in sensible terms, and we call it the Monster model) and we assume that all parameter sets are subsets of $\mathbb{M}$. So we can always think of a complete type as being the type of an element of $\mathbb{M}$ over a subset of $\mathbb{M}$.