Cellular covers of groups and modules

Lutz Strüngmann

Abstract

Cellular covers are the algebraic counterpart of cellular approximations of topological spaces in the sense of J.H.C. Whitehead, or the more general cellularization maps extensively studied in homotopy theory in the 90s. In some cases there is even a good interplay between cellularization of spaces and cellularization of groups and modules, as was previously known for the dual case, namely for localizations. This was indeed the main reason for looking carefully at the algebraic counterpart. Recall that a homomorphism $\pi : G \to H$ of groups is a *cellular cover* over H, if every homomorphism $\varphi : G \to H$ lifts uniquely to an endomorphism $\widetilde{\varphi}$ of G such that $\pi\widetilde{\varphi} = \varphi$. In case that $\pi : G \to H$ is an epimorphism we say that

$$1 \to K \to G \xrightarrow{\pi} H \to 1$$

is a *cellular exact sequence*. The main objective is to classify (up to isomorphism) all possible cellular exact sequences with either fixing the cokernel H or the kernel K.
In this talk we give an overview over recent results measuring the success or failure of a classification of cellular covers. This includes constructions of cellular covers as well as structure theorems. For instance, G is abelian, nilpotent, or an R-module, whenever H is abelian, nilpotent or an R-module, respectively. However, if H is reduced and torsion, then the cellular sequence collapses while if K is a cotorsion-free module, then there exist arbitrarily large cellular covers G with K as a kernel.