General Relativity
Seminar

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Abstract: Introduced in 1915 by Einstein, General Relativity is a geometric theory of gravitation. It generalizes Special Relativity and Newton's Law of Gravitation. Its essence are Einstein's Field Equations, which describe how spacetime is curved by the presence of energy and matter. The goal of this seminar is to derive and discuss Einstein's Field Equations. Among the topics to be discussed are: (1) Newtonian limit and weak gravitation, (2) the Schwarzschild solution and predictions for the motions of planets and the bending of light, (3) the extended Schwarzschild solution and black holes, (4) the Friedmann-Lemaitre-Robertson-Walker solutions and explanation of cosmological redshift and the origins of big-bang.

References:

1. Sean Carroll: "Spacetime and geometry: an introduction to general relativity" (2014, 2nd edition) + his lecture notes from MIT,
`http://preposterousuniverse.com/spacetimeandgeometry/`

2. Bernard Schutz: "A first course in general relativity" (2nd edition, 2014)

3. Steven Weinberg: "Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity" (1972)

4. Walter Isaacson: "Einstein: His Life and Universe" (2007)

Talks:

1. **20.10.2015:**
   Pre-relativistic notions of spacetime structure
   (Newton, Leibniz, Mach)

   Newton's theory of gravitation; invariance under Galileo group; Principle of Galilean Relativity.

   Separation of space and time; the idea of simultaneity.

   Bibliography: Weinberg (Chapter 1), Schutz (1.1), Carroll (1.1 and 1.2)

2. **27.10.2015:**
   Special Relativity and Flat Spacetime, I: Minkowski spacetime

   Minkowski space: four-vector; metric $\eta_{\mu\nu}$; Einstein summation convention; timelike, null, spacelike vectors; curves in Minkovski spacetime and four-velocity; light cone; spacetime diagrams; spacetime interval, proper time, proper time and coordinate time are different; time dilation; the twin paradox.

   Lorentz transformations; spacetime interval is invariant under Lorentz transformations; Principle of invariance of speed of light (Einstein).

   Bibliography: Weinberg (Chapter 2); Carroll (Chapter 1), Schutz (Chapter 1).

3. **03.11.2015:**

   Special Relativity and Flat Spacetime, II: Relativistic kinematics and particle dynamics

   Particles in Special Relativity; The four-velocity vector; inertial frame; accelerated particle; comoving frame; the four-momentum vector; energy; conservation of four-momentum.

   Photons.

   Bibliography: Schutz (Chapter 2), Weinberg (Chapter 2), Carroll (1.9).

4. **10.11.2015:**

   Manifolds, I

   Gravity as Geometry; motivation why we need to consider manifolds

   Abstract Manifolds;
   Tangent space: tangent vector via curves and via derivations; coordinate expression;
   Vector fields and Lie brackets;

   Tensors; The metric tensor; signature

   Euclidean and Minkowski space as abstract manifolds with metrics.

   Bibliography: Carroll (Chapter 2: 2.1-2.5);

5. **17.11.2015:**

   Manifolds, II

   Covariant derivative, Christoffel symbols, covariant derivative of a tensor

   Levi-Civita connection.

   Parallel transport and geodesics

   Bibliography: Carroll (3.1-3.3), Schutz (Chapter 6)

6. **24.11.2015:**

   Manifolds, III: The Riemannian curvature tensor

   Curvature: three equivalent definitions

   (a) lack of commutativity of $\nabla_X \nabla_Y$ and $\nabla_Y \nabla_X$

   (b) via parallel transport

   (c) via geodesic deflection.

   The Bianchi identities

   Ricci curvature and scalar curvature

7. **01.11.2015:**

   Physics in a curved spacetime

   (a) Lorentzian manifolds: light cone, null-vectors, types of geodesics, Simultaneity: spacelike hypersurfaces

   (b) Symmetries and Killing vectors

   (c) Physics in curved spacetime: Einstein's Equivalence Principle

   Bibliography: Carroll (3.8, 4.1); Schutz (chapter 7); Weinberg (3.1 and 3.2)

8. **08.12.2015:**

   Einstein's Equation

   Derivation of Einstein's Equation; Einstein's Equation in vacuum.

   Bibliography: Carroll (4.2, 4.7); Schutz (8.1 and 8.2); Weinberg (Chapter 7)

9. **15.12.2015:**

   The Schwarzschild Solution, I

   The Schwarzschild metric
   Birkoff's Theorem: the Schwarzschild metric is the unique vacuum solution with spherical symmetry
   Singularities

   Bibliography: Carroll (5.1 -5.3); Weinberg (8.1 and 8.2)

10. **12.01.2016:**

    The Schwarzschild Solution, II

Geodesics
Experimental tests: precession of periphelia; the gravitational red shift; deflection of light by the sun.

Bibliography: Carroll (5.4-5.5); Schutz (Chapter 11), Weinberg (8.5 and 8.6)

11. **19.01.2016:**

    Perturbation Theory and Gravitational Radiation

    Linearized Einstein's Equation; Gauge Transformations; Gravitational wave solutions; Production of gravitational waves; Energy loss due to gravitational radiation

    Bibliography: Carroll (7.1, 7.4-7.6); Schutz (8.3 and Chapter 9); Weinberg (7.6)

12. **26.01.2016:**

    Cosmology, I

    Maximally Symmetric Universes: de Sitter and anti-de Sitter spaces
    Robertson-Walker metrics
    The Friedmann Equation: the Hubble parameter, Hubble time,

    Bibliography: Carroll (8.1, 8.2, 8,3); Schutz (Chapter 12)

13. **02.02.2016:**

    Cosmology, II

    Evolution of the scale factor: singularities and Big Bang
    Red shifts and distances
    Our universe

    Bibliography: Carroll (8.4, 8.5, 8.7); Schutz (Chapter 12)

14. **09.02.2016:**

    Black holes: The Schwarzschild Black Hole

    Black Holes in Newtonian gravity
    The Schwarzschild black hole: event horizon
    Inside the black hole: the maximally extended Schwarzschild solution
    Stars and Black Holes

    Bibliography: Carroll (5.6-5.8); Schutz (Chapter 11).