MA5P8 CONFORMAL FIELD THEORY

Section 1: The Virasoro algebra

1. Recall the definition of a conformal map.

2. Check that $\varphi(\xi) = e^{i\xi}$ defines a conformal map $\varphi\colon \mathbb{C} \to \mathbb{C}^*$ with respect to the metric $g(v,w) = \frac{1}{2}(\overline{v} \cdot w + v \cdot \overline{w})$ on $\mathbb{C}$ and $\mathbb{C}^*$.

3. For each of the following vector fields X determine a family of conformal maps on the complex plane, such that X is the associated infinitesimal conformal transformation:

$$\partial_z + \partial_{\overline{z}}, \quad i\partial_z - i\partial_{\overline{z}}, \quad iz\partial_z - i\overline{z}\partial_{\overline{z}}, \quad z\partial_z + \overline{z}\partial_{\overline{z}}.$$

   Recall the geometric interpretation for each of these families.

4. Remind yourself of the definition of the Witt algebra, in particular its relation to conformal maps on the plane, its standard generators, and the commutator relation for them. List the generators of the infinitesimal Möbius transforms.

5. Check that the Jacobi identity holds for the Witt algebra and for one of its non-trivial central extensions.