MA5P8 CONFORMAL FIELD THEORY

Section 3: Free bosonic theories

1. Remind yourself of the definitions of the Fock space representation of the free boson algebra (FFB) and the Virasoro modes in terms of the FFB.

2. Calculate $[L_n, L_m]$ for $m+n=0$ and $[L_n, a_k]$ in the FFB, using the definitions given in the previous question.

3. Show that if $[L_n, L_m]\Psi = \left\{(m-n)L_{m+n} + \frac{c}{12}\delta_{m+n,0}m(m^2-1)\right\}\Psi$ for a state Ψ in the Fock space then also $[L_n, L_m]a_k\Psi = \{(m-n)L_{m+n} + \frac{c}{12}\delta_{m+n,0}m(m^2 -1)\}\, a_k\Psi$ for all a_k.

4. Show that in the FFB $B^{(0)}$, all $a_1^m|0\rangle$ are lwvs.

5. Let $F_\pm^{(0)}$, $F^{(1/16)}$ denote the three irreducible unitary representations of the Virasoro algebra at $c=\frac{1}{2}$ with lowest weights 0, $\frac{1}{2}$, $\frac{1}{16}$, respectively. Consider $\mathbb{H} := \mathbb{H}^+ \oplus \mathbb{H}^- \oplus \mathbb{H}^0$ with

$$\begin{aligned}
\mathbb{H}^+ &:= \left[(F_+^{(0)})^{\otimes 2} \oplus (F_-^{(0)})^{\otimes 2}\right] \otimes \left[(\overline{F}_+^{(0)})^{\otimes 2} \oplus (\overline{F}_-^{(0)})^{\otimes 2}\right],\\
\mathbb{H}^- &:= \left[\left(F_+^{(0)} \otimes F_-^{(0)}\right) \oplus \left(F_-^{(0)} \otimes F_+^{(0)}\right)\right] \otimes \left[\left(\overline{F}_+^{(0)} \otimes \overline{F}_-^{(0)}\right) \oplus \left(\overline{F}_-^{(0)} \otimes \overline{F}_+^{(0)}\right)\right],\\
\mathbb{H}^0 &:= \left[F^{(1/16)}\right]^{\otimes 2} \otimes \left[\overline{F}^{(1/16)}\right]^{\otimes 2}.
\end{aligned}$$

This is a unitary representation of

$$\mathcal{V}\mathrm{ir}_{\frac{1}{2}} \oplus \mathcal{V}\mathrm{ir}_{\frac{1}{2}} \oplus \overline{\mathcal{V}\mathrm{ir}_{\frac{1}{2}}} \oplus \overline{\mathcal{V}\mathrm{ir}_{\frac{1}{2}}} \cong \mathcal{V}\mathrm{ir}_1 \oplus \overline{\mathcal{V}\mathrm{ir}_1}.$$

Show that the partition function Z of this representation agrees with the partition function of a free boson compactified on a circle of radius $R=\sqrt{2}$.

[Hint: See exercise 6 on worksheet 2.]