MA5P8 CONFORMAL FIELD THEORY

Section 4: The operator product expansion

1. Remind yourself of the meaning of the expression
$$\left[\oint dz\, A(z), \oint dw\, B(w)\right]$$
for holomorphic fields A, B in a CFT, relating it to radial ordering.

2. Consider a Virasoro field T at central charge c and the fields $j(z) = \sum_n a_n z^{n-1}$ and $\psi(z) = \sum_k \psi_k z^{k-\frac{1}{2}}$ with a_n the modes of the free boson algebra and ψ_k the modes of the free fermion algebra in the NS sector. Show that the OPEs
$$T(z)\,T(w) \;\sim\; \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \text{reg.}$$
and
$$j(z)j(w) \sim \frac{1}{(z-w)^2} + \text{reg.}, \qquad \psi(z)\psi(w) \sim \frac{1}{z-w} + \text{reg.} \sim -\psi(w)\psi(z)$$
encode the known commutator or anti-commutator relations for the modes.

3. Let $L_n^{(1)}$ denote the modes of the Virasoro algebra at central charge $c^{(1)} = 1$ used in the FFB. Consider the corresponding Virasoro field $T^{(1)}(z) = \sum_n L_n^{(1)} z^{n-2}$, and let $j(z)$ be as in the previous question. Check that for $\alpha \in \mathbb{R}$ the field $T(z) := T^{(1)}(z) + \sqrt{2}\alpha\partial j(z)$ is a Virasoro field with central charge $c = 1 - 24\alpha^2$.

4. Show that in a unitary lowest weight representation of the Virasoro algebra of central charge $c = 0$ with ground state $|0\rangle$ of weight 0 one has $L_n|0\rangle = 0$ for all $n \in \mathbb{Z}$.