PROBLEM . 1 .

$$\left(\begin{array}{ccc|c} 3 & -5 & -4 & b_1 \\ -3 & -2 & 4 & b_2 \\ 6 & 1 & 8 & b_3 \end{array}\right)$$

$$\xrightarrow[-2r_1+r_3]{r_1+r_2} \left(\begin{array}{ccc|c} 3 & -5 & -4 & b_1 \\ 0 & -7 & 0 & b_1+b_2 \\ 0 & 11 & 16 & b_3-2b_1 \end{array}\right)$$

$$\xrightarrow{-1/7\, r_2} \left(\begin{array}{ccc|c} 3 & -5 & -4 & b_1 \\ 0 & 1 & 0 & -1/7\,(b_1+b_2) \\ 0 & 11 & 16 & b_3-2b_1 \end{array}\right)$$

$$\xrightarrow{-11r_2+r_3} \left(\begin{array}{ccc|c} 3 & -5 & -4 & b_1 \\ 0 & 1 & 0 & -1/7\,(b_1+b_2) \\ 0 & 0 & 16 & 11/7\,(b_1+b_2)+b_3-2b_1 \end{array}\right)$$

FOR $b_1 = 7,\ b_2 = -1,\ b_3 = -4$:

$$\left(\begin{array}{ccc|c} 3 & -5 & -4 & 7 \\ 0 & 1 & 0 & -1/7\,(6) \\ 0 & 0 & 16 & 11/7\,(.\,6\,..) + \cancel{(-4)} - 2(7) \end{array}\right)$$

$$x_3 = \left(\frac{11}{7}(6) - 8\right)\cdot\frac{1}{16} =: a$$

$$x_2 = -6/7, \quad 3x_1 = 5\left(-\frac{6}{7}\right) + 4a + 7$$

$$\therefore x_1 = \left(\frac{19}{7} + 4a\right)\cdot\frac{1}{3}$$

PROBLEM . 2 .

(a) $z_1 z_2 = (2+i)(1+3i)$

$= 2 + 6i + i - 3 = -1 + 7i$

$$z_2^{-1} = \frac{1}{1+3i} = \frac{1-3i}{1+9} = \frac{1}{10}(1-3i)$$

$$|z_1| = \sqrt{4+1} = \sqrt{5}$$

(b) $z = 3(1+i) = 3e^{i(\pi/4)}$

$$\Rightarrow z^8 = \left(3e^{i\pi/4}\right)^8 = 3^8 \underbrace{e^{i(2\pi)}}_{=1} = 3^8$$

PROBLEM . 3 .

(a) $\dfrac{dy}{dt} = \dfrac{yt}{t^2+1}, \quad y(t=0) = 1$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{t}{t^2+1}\,dt$$

$$\Rightarrow \ln(y) = \frac{1}{2}\ln(t^2+1) + c$$

$$\therefore \ln\left(\frac{y}{(t^2+1)^{1/2}}\right) = c$$

$$\therefore y = e^c (t^2+1)^{1/2}$$

$y(0) = 1 \Rightarrow 1 = e^c$

SO THE SOLUTION IS : $\underline{\underline{y = (t^2+1)^{1/2}}}$

(b) $\frac{dy}{dt} + y = e^{-t}$ (*)

EITHER USE YOUR FORMULA :

$$y(t) = y_0 e^{-A(t)} + e^{-A(t)} \int \ldots$$

$$, A(t) := \int_{t_0}^{t} a(\tau)\, d\tau$$

(WHERE $y' + a(t)\, y = b(t)$.)

OR MULTIPLY BOTH SIDES OF (*) BY INTEGRATING FACTOR e^{t} TO GET

$$\frac{d}{dt}\left(y e^{t}\right) = 1$$

$$\therefore \quad y e^{t} = t + c \Rightarrow y = t e^{-t} + c e^{-t}$$

$$y(0) = 1 \Rightarrow 1 = c$$

SO THE SOLUTION IS : $\underline{\underline{y = t e^{-t} + e^{-t}}}$.

(c) $y'' + 4y' + 3y = 0$ SUBSTITUTE $y = e^{\lambda t}$:

$$\lambda^2 + 4\lambda + 3 = 0 \Rightarrow (\lambda + 3)(\lambda + 1) = 0$$

$$\lambda_1 = -3, \quad \lambda_2 = -1$$

$$y(t) = c_1 e^{-3t} + c_2 e^{-t}$$

$$\therefore \quad y'(t) = -3c_1 e^{-3t} - c_2 e^{-t}$$

$$y(0) = 1 \rightarrow 1 = c_1 + c_2$$

$$y'(0) = 0 \rightarrow 0 = -3c_1 - c_2$$

$$\therefore \quad 1 = -2c_1 \Rightarrow c_1 = -\tfrac{1}{2}$$

$$\Rightarrow c_2 = \tfrac{3}{2}$$

SO THE SOLUTION IS :

$$\underline{\underline{y(t) = -\tfrac{1}{2} e^{-3t} + \tfrac{3}{2} e^{-t}}}$$

PROBLEM . 4 .

$$u' = (2-u)(2+u)$$

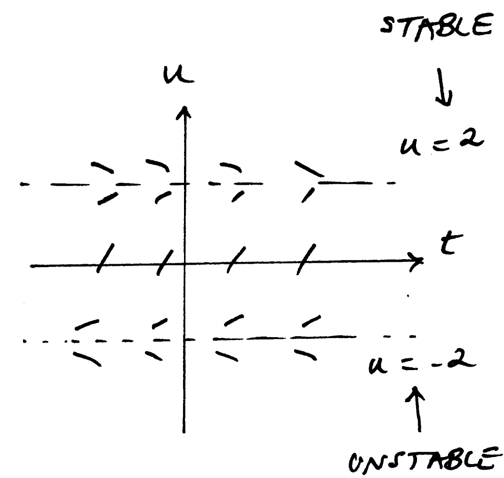
$$u' > 0$$

$$\Updownarrow$$

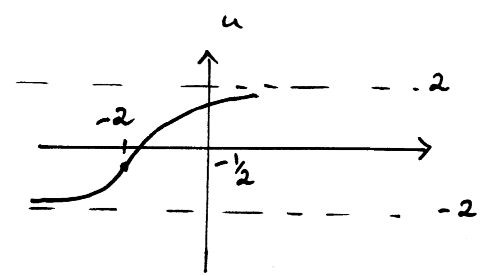
$$\begin{cases} (2-u) > 0, & (2+u) > 0 \\ (2-u) < 0, & (2+u) < 0 \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} u < 2, & u > -2 \quad \checkmark \\ 2 < u, & u < -2 \quad \text{NO SOLUTIONS} \end{cases}$$



$u(-2) = -\frac{1}{2}$:



$u(0) = 2$: