ABSTRACTS OF TALKS

Alessandro Andretta
Descriptive set theory and the density point property

Let (X, d) be Polish, μ a Borel measure on X that does not vanish on nonempty open sets, and $A \subseteq X$ a measurable set. The density of a point $x \in X$ with respect to A is

$$\mathscr{D}_A(x) = \lim_{\varepsilon \to 0^+} \frac{\mu(A \cap \mathrm{B}(x;\varepsilon))}{\mu(\mathrm{B}(x;\varepsilon))},$$

where $\mathrm{B}(x;\varepsilon)$ is the ball centered in x of radius ε, and let

$$\Phi(A) = \{x \in X \mid \mathscr{D}_A(x) = 1\}.$$

(X, d, μ) has the density point property (DPP) if $\Phi(A) \triangle A$ is null, for all measurable A. The Lebesgue density theorem says that $\mathbb{R}$ with the usual metric and the Lebesgue measure has the density point property. The DPP depends heavily on the metric d — not all (X, d, μ) satisfy it.

I will survey some recent results on these matters, and on the descriptive complexity of $\mathrm{ran}(\mathscr{D}_A)$ when the ambient space is $\mathbb{R}$ or the Cantor space ${}^{\omega}2$.

This is joint work with Riccardo Camerlo (Politecnico di Torino) and Camillo Costantini (Università di Torino).

Andreas Blass
Partition Properties of Generic Ultrafilters

When an ultrafilter over the natural numbers is produced by forcing, it often has some partition properties, depending, of course, on the particular notion of forcing. In some situations, there is a converse, so that partition properties imply genericity. I'll begin the talk by describing these matters in the long-known case of forcing with infinite subsets of ω (dual to the cofinite filter). Then I'll develop part of an analogous theory when the cofinite filter is replaced by its tensor square (also called its Fubini square). Finally, I'll describe a possibly surprising connection between these two sorts of generic ultrafilters.

Natasha Dobrinen
Generalized Ellentuck spaces

The partial ordering $\mathcal{P}(\omega \times \omega)/(\mathrm{Fin} \times \mathrm{Fin})$ forces a generic ultrafilter $\mathcal{G}_2$ which is not a p-point but is Rudin-Keisler minimal above its projected Ramsey ultrafilter. In [1], it was shown that $\mathcal{G}_2$ is not maximum in the Tukey order of ultrafilters, yet is also not basically generated. However, it was left open whether $\mathcal{G}_2$ is Tukey minimal over its projected Ramsey ultrafilter.

In [2], we prove that $\mathcal{G}_2$ is indeed Tukey minimal over its projected Ramsey ultrafilter. Furthermore, we prove that for each $k \geq 2$, the ultrafilter $\mathcal{G}_k$ forced by $\mathcal{P}(\omega^k)/\mathrm{Fin}^{\otimes k}$ has initial Tukey structure exactly a chain of length k. These are the first results finding initial Tukey structures for non-p-points. The proof proceeds by extracting a dense subset $\mathcal{E}_k$ from $(\mathrm{Fin}^{\otimes k})^+$, finding the correct way to define the finite approximations, and proving it to be a topological Ramsey space. In fact, these spaces $\mathcal{E}_k$ form a hierarchy naturally generalizing the Ellentuck space, and yet are the first topological Ramsey spaces which have essentially infinite blocks instead of finite blocks.

New Ramsey-classification theorems for equivalence relations on fronts on $\mathcal{E}_k$ are proved, extending the Pudlák-Rödl Theorem for fronts on the Ellentuck space. These classification theorems give exact irreducible maps canonizing equivalence relations, in contrast to other recently constructed topological Ramsey spaces, where there can be more than one inner Nash-Williams function canonizing an equivalence relation. Applying these theorems, we find the initial Tukey structure below $\mathcal{G}_k$.

References

[1] Andreas Blass, Natasha Dobrinen, and Dilip Raghavan, *The next best thing to a p-point*, (2013), Submitted.

[2] Natasha Dobrinen, *Generalized Ellentuck spaces and applications in the Tukey types of non-p-points*, Preprint.

Mirna Džamonja
Combinatorial versions of SCH

Much deep and intricate work in set theory has centered around the singular cardinal hypothesis, deciding to what extent it is true. We are interested in investigations of further combinatorial properties of singular cardinals and their successors, not limited to the size of their power set, and in knowing to what extent ZFC determines these properties. For example, is it possible to have a model of set theory in which there is no universal graph on $\aleph_\omega^+$?

Vera Fischer
Template iterations

A well-known cardinal invariant of the real line is the minimum size of a subgroup of the group of all permutations of the natural numbers, which has the property that all of its non-identity elements have only finitely many fixed points and which is maximal (with respect to this property) under inclusion. Another well-known invariant, known as the dominating number, is defined as the minimum size of a family F of reals (e.g. functions from the natural numbers to the natural numbers) with the property that every real is eventually dominated by an element of the family F.

The independence of the dominating number and the minimal size of a maximal cofinitary group was obtained only after a ground-breaking work of Shelah and the appearance of his "template iteration" forcing technique. We further develop these techniques to show that the minimum size of a maximal cofinitary group, as well as some of its relatives, can be of countable cofinality. In addition we will discuss other recent developments of the subject and conclude with some open questions.

Sy-David Friedman
The Δ_1-Definability of the Nonstationary Ideal

Mekler-Shelah (with a correction by Hyttinen-Rautila) showed that the nonstationary ideal on ω_1 can be Δ_1-definable. This was generalised by Hyttinen-Rautila, Friedman-Hyttinen-Kulikov and Friedman-Wu to show that various restrictions of the nonstationary ideal on a regular cardinal greater than ω_1 can be Δ_1-definable. Friedman-Wu-Zdomskyy showed that the unrestricted nonstationary ideal on any successor cardinal can be Δ_1-definable. Recent work of mine with Stefan Hoffelner shows that the nonstationary ideal on ω_1 restricted to a costationary set can be made both Δ_1-definable and saturated using one Woodin cardinal. Woodin showed that the unrestricted nonstationary ideal on ω_1 can be both saturated and Δ_1-definable using ω many Woodin cardinals; it is open if one Woodin cardinal suffices. This work has consequences for generalised descriptive set theory, as stationary restrictions of the nonstationary ideal do not have the Baire property.

Martin Goldstern
More cardinals in Cichoń's diagram

In a recently submitted paper (coauthors: Arthur Fischer, Jakob Kellner, Saharon Shelah) we showed that these 5 cardinals can be different:

$\aleph_1 = \mathfrak{d} = \mathrm{cov(null)}$, non(meager), non(null), cf(null), $2^{\aleph_0}$.

The construction is a (kind of) countable support product of creature forcing notions. I will present some aspects of this construction, and also mention some extensions (which are still work in progress):

- Adding the reaping number as a sixth distinct value.
- Adding random reals to increase cov(null).

Menachem Kojman
Density versus covering

The finite binomial function $\binom{n}{k}$ generalizes to the following two functions in the infinite case: the covering number $Cov(\lambda, \kappa)$ and the density $\mathcal{D}(\lambda, \kappa)$. These functions obey different constraints and their behavior will be compared.

Péter Komjáth
Some chromatic number and Ramsey results for triple systems

We survey some recent results on obligatory (finite) subsystems of uncountably chromatic triple systems and some connected Ramsey type problems of Erdős, Galvin, and Hajnal.

Giorgio Laguzzi
Generalized Silver measurability

The talk concerns the study of tree-like forcings and the related regularity properties for the generalized Cantor and Baire spaces, with a particular emphasis on the Silver forcing. A result of Shelah and Halko shows that there exists a $\mathbf{\Sigma}^1_1$ set without the Baire property (i.e., the club filter), and this makes the situation different from the standard setting. The key point is that in the generalized setting we do not have an analogue of the factor lemma. Nevertheless, in some cases we can recover a partial version of it, and prove that sufficiently many reals have good quotients. In particular we will prove that an iteration of κ-Cohen forcing forces that all projective subsets of 2^κ are stationary Silver measurable.

Menachem Magidor
The Set Theory of generalized Logics

A generalized logic is a mechanism of extending first order logic in order to express properties of mathematical structures or of elements of such structures. A prime example is higher order logic, like second order logic. A generalized logic is typically sensitive to the set theoretical frame work in which the mathematical structure is embedded. This creates an interesting interplay between properties of the logic, like compactness, Skolem-Löwenheim theorems etc. and the underlying Set Theory.

The interaction can go both ways: A desired properties of the logic under consideration can be used as a motivation for new axioms for Set Theory, for new notions of large cardinals, etc. On the other hand analysis of the possible properties of a given logic in different set theoretic universes can give some insight into the the strength of the logic and its relations with other logics. For instance a typical (admittedly vague) problem is the extent by which a given logic is really logic, or is it Set Theory in disguise.

In this talk we shall show some examples of such interplay.

Adrian Mathias
Provident Set Theory

I review

- the definitions of "rudimentary recursion", "gentle function" and "provident set";
- the main lemma in the proof of Bowler's result that the class of gentle functions is closed under composition;
- the proof of my result that every infinite level (including the successor ones) of the Gödel hierarchy is closed under the Scott-McCarty pairing and unpairing functions;

and some other related problems and results.

Janusz Pawlikowski
Playing with countable support of Mathias forcing

I am going to discuss combinatorial principles in the spirit of the Covering Property Axiom that hold in the Mathias model and are expressible in terms of games using Borel sets and functions. They are designed to give "the combinatorial core" of the Mathias model and to imply via a straightforward deduction the result of Shelah and Spinas that the distributivity of the algebra $\mathcal{P}(\mathbb{N})/\mathrm{Fin}$ differs from that of its square.

This is a joint work with Wojciech Stadnicki.

Lajos Soukup
Monochromatic paths and path squares in infinite graphs

Our goal is to find nice partitions of edge-colored infinite graphs. In particular, we are interested in partitioning the vertices of complete or nearly complete graphs into monochromatic paths and powers of paths.

An *r-edge coloring* of a graph (or hypergraph) $G = (V, R)$ is a map $c : E \to r$ where r is some cardinal. Investigations began in the 80s with a results of Rado implying that the every r-edge colored ($r \in \omega$) complete graph on ω can be partitioned into r monochromatic paths with different colors.

We extend this result for hypergraphs by proving that every r-edge colored complete n-uniform hypergraph on ω can be partitioned into r monochromatic tight paths with different colors.

We also proved that for each natural number r and m there is a natural M such that the every r-edge colored complete graph on ω can be partitioned into M monochromatic m^{th} power of paths apart from a finite set. Using a recent result of Pokrovskiy on finite graphs we could show that every 2-edge color complete graph on ω can be partitioned into 5 squares of a path.

Finally we could generalize Rado's result for some uncountable graphs: every 2-edge colored infinite complete graph on ω_1 can be partitioned into 2 monochromatic paths.

This is a joint work with Márton Elekes, Dániel Soukup and Zoltán Szentmiklóssy.

Stevo Todorčević
Algebras that are hereditarily interval

We investigate the structure of Boolean algebras which have the property that all of their subalgebras are generated by subsets that are totally ordered by the inclusion. We give a generalization of a classical result of Mostowski and Tarski asserting that all countable Boolean algebras belong to this class. This is a joint work with M. Bekkali.

Boban Veličković
Iteration of semiproper forcing revisited

We present a variation of the method for iterating proper forcing using finite chains of elementary sub models as side conditions, introduced by Itay Neeman several years ago. This approach is quite flexible and allows us to iterate large classes of non proper forcing notions, such as semiproper forcings. A related, but different approach was recently proposed by Gitik and Magidor. We also apply this method to give a simple proof of the consistency of PFA together with the non stationary ideal being non precipitous, a result that was previously obtained by different methods by Shelah and the author.

Matteo Viale
Generic absoluteness results as steps towards a "complete" axiom system for set theory

We shall present several generic absoluteness results regarding the theory of $H_{\aleph_2}$, and also some conjectures regarding possible strengthenings of these results. This is part of a thread of researches which I'm undertaking also with Audrito and Ikegami. Relevant preprints outlining in all details these results are available on my webpage http://www.personalweb.unito.it/matteo.viale/.

-Absoluteness via Resurrection (with Giorgio Audrito)

-Category forcings, MM^{+++}, and generic absoluteness for strong forcing axioms

-Martin's maximum revisited

Philip Welch
Determinacy of some pointclasses just beyond co-analytic

It is well known that infinite perfect information two person games at low levels in the arithmetic hierarchy of sets have winning strategies for one of the players, and moreover this fact can be proven in analysis alone. This has led people to consider reverse mathematical analyses of precisely which subsystems of second order arithmetic are needed. It is possible to lift such arguments to establish the amount of determinacy, properly including analytic determinacy, provable in the corresponding theory fragments of ZF^-+ "there is a measurable cardinal" (the latter theory corresponding roughly to 'analysis' beyond Π^1_1).

Recently Montalban and Shore gave a precise delineation of the amount of determinacy provable in analysis. This too should lift to our context. We summarise some recent joint work with Chris Le Sueur in this direction.

Lyubomyr Zdomskyy
Between Polish and Completely Baire

We will consider the following properties of a separable metrizable space X:
(1) X is Polish;
(2) For every countable crowded subset Q of ${}^{\mathbb{R}}X$ there exists a crowded subset Q_0 of ${}^{\mathbb{R}}Q$ with compact closure;
(3) Every closed subspace of X is either scattered or it contains a homeomorphic copy of the Cantor space;
(4) Every closed subspace of X is a Baire space.

While (4) is the well-known property of being completely Baire, properties (2) and (3) have been recently introduced by Kunen, Medini and Zdomskyy, who named them the Miller property and the Cantor-Bendixson property, respectively. It turns out that the implications (1) $\to$ (2) $\to$ (3) $\to$ (4) hold for every space X. Furthermore, it follows from a classical result of Hurewicz that all these implications are equivalences if X is co-analytic. Under the axiom of Projective Determinacy, this equivalence result extends to all projective spaces. We will complete the picture by giving a ZFC counterexample and a consistent definable counterexample of lowest possible complexity to the implication $(i) \leftarrow (i+1)$ for $i = 1; 2; 3$.

Based on a joint work with Andrea Medini.