

## No Borel Connections for the Unsplitting Relations

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**Abstract.** We prove that there is no Borel connection for non-trivial pairs of unsplitting relations. This was conjectured in [3].

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VOJTÁŠ [6] introduced a framework in which cardinal characteristics of the continuum can be regarded as norms of corresponding relations  $\mathbf{A} = (A_-, A_+, A)$  with  $A_-, A_+ \subseteq 2^\omega$ ,  $A \subseteq A_- \times A_+$ , and the norm

$$\|\mathbf{A}\| = \min\{|\mathcal{Z}| : \mathcal{Z} \subseteq A_+ \wedge (\forall x \in A_-) (\exists z \in \mathcal{Z}) A(x, z)\}.$$

A *Galois-Tukey connection* from a relation  $\mathbf{B}$  to a relation  $\mathbf{A}$ , which we call as in [1] a *morphism from  $\mathbf{A}$  to  $\mathbf{B}$* , is a pair of functions  $(\alpha, \beta)$  such that

$$\begin{aligned} \alpha : B_- &\longrightarrow A_-, & \beta : A_+ &\longrightarrow B_+, \\ (\forall b \in B_-) (\forall a \in A_+) (A(\alpha(b), a) &\rightarrow B(b, \beta(a))). \end{aligned}$$

If there is a morphism from  $\mathbf{A}$  to  $\mathbf{B}$ , then  $\|\mathbf{B}\| \leq \|\mathbf{A}\|$ , and indeed the proofs of the inequalities usually exhibit morphisms between the corresponding relations.

We deal with the unsplitting relations: For  $n \geq 1$ , we have

$$\begin{aligned} \mathbf{R}_n &= (n^\omega, [\omega]^\omega, \{(f, Y) : f \text{ is almost constant on } Y\}), \\ \mathbf{R}_n^\sharp &= (n^\omega, [\omega]^\omega, \{(f, Y) : f \text{ is constant on } Y\}), \\ \mathfrak{r}_n &= \|\mathbf{R}_n\|, & \mathfrak{r}_n^\sharp &= \|\mathbf{R}_n^\sharp\|. \end{aligned}$$

( $f$  is almost constant on  $Y$  if there is a finite  $Y_0 \subseteq Y$  such that  $f$  is constant on  $Y \setminus Y_0$ .)  $\mathfrak{r}_2$  is the usual unsplitting number. It is easy to see that  $\mathfrak{r}_m = \mathfrak{r}_n$  for  $m, n \geq 2$ .

In [3] it was proved that there is no morphism for the sharp unsplitting relations with Baire measurable first component, and it was conjectured that the same is true for the ordinary unsplitting relations. In this paper we prove this conjecture, under the additional premise that  $\alpha$  and  $\beta$  are Lebesgue measurable.

The present work is built partly upon techniques from the mentioned work on the sharp unsplitting relations, to which we add some additional steps mainly coming from

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descriptive set theory. First we recall the (well-known) notions of measurability that we use. Let  $\mathcal{X}$  and  $\mathcal{Y}$  be topological spaces. A function  $f : \mathcal{X} \rightarrow \mathcal{Y}$  is called *Borel measurable* if for every open set  $O$  in  $\mathcal{Y}$ ,  $(f^{-1})''(O)$  is in the Borel  $\sigma$ -algebra of  $\mathcal{X}$ . For a measure  $\mu$  on  $\mathcal{X}$ ,  $f : \mathcal{X} \rightarrow \mathcal{Y}$  is called  *$\mu$ -measurable* iff, for every open set  $O$  in  $\mathcal{Y}$ ,  $(f^{-1})''(O)$  is in the domain of  $\mu$ . Finally,  $f : \mathcal{X} \rightarrow \mathcal{Y}$  is called *Baire measurable* iff for every open set  $O$  in  $\mathcal{Y}$ ,  $(f^{-1})''(O)$  is Baire measurable, i. e., there is an open subset  $O'$  of  $\mathcal{X}$  such that  $(f^{-1})''(O) \Delta O'$  is meager in  $\mathcal{X}$ . In our application,  $\mathcal{X}$  will be  ${}^\omega n$  for some natural  $n \geq 2$ , equipped with the product topology and the usual Haar measure which we call  $\mu$ , or it will be the real interval  $[0, 1]$  with its usual topology and the Lebesgue measure.  $\mathcal{Y}$  will be  ${}^\omega m$  for some natural  $m \geq 2$ , equipped with the product topology, or it will be again the real interval  $[0, 1]$  with its usual topology. Any Borel measurable  $\alpha : {}^\omega n \rightarrow {}^\omega m$  is Baire measurable and  $\mu$ -measurable. Similarly any Borel measurable  $\beta : [0, 1] \rightarrow [0, 1]$  is Lebesgue measurable. So the following theorem, which is the main and only result of this paper, implies that there are no Borel connections for non-trivial pairs of unsplitting relations, and hence answers a question of BLASS' [1].

**THEOREM 1.**<sup>3)</sup> *For  $n > m$ , there is no morphism  $(\alpha, \beta)$  with a Baire measurable function  $\alpha$  and  $\mu$ -measurable  $\alpha$  and Lebesgue measurable  $\beta$  from  $\mathbf{R}_m$  to  $\mathbf{R}_n$ .*

The proof is preceded by the explanation of some notation.

**NOTATION.** We give some notation concerning trees, finite sequences, etc. The symbols  $\subset$  and  $\supset$  denote the proper relations.  ${}^{\omega>}D = \{t : (\exists h \in \omega) t \restriction h \in D\}$ .  $[D]^\omega = \{X \subseteq D : |X| = \omega\}$ .  $T \subseteq {}^{\omega>}n$  is a *tree*, if for all  $t \in T$  and for all  $s \in {}^{\omega>}n$ , if  $s \subseteq t$ , then  $s \in T$ . The  *$h$ -th level of a tree  $T$*  is  $T \cap {}^h\omega$ . We write  $T \restriction h$  for  $T \cap {}^{h>}\omega$ . The *height of the tree  $T \subseteq {}^{\omega>}n$* ,  $\text{ht}(T)$ , is the smallest  $h \in (\omega + 1)$  such that  $T \subseteq {}^{h>}n$ .  $T$  is an  *$\omega$ -tree* if  $T \neq \emptyset$  and for all  $s \in T$  there is  $t \in T$  such that  $t \supset s$ .

Let  $T$  be an arbitrary (not necessarily  $\omega$ -) tree. A node  $t \in T \subseteq {}^{\omega>}n$  is a *branching point of  $T$*  if there are  $c_0 \neq c_1 \in n$  such that  $t \hat{\ } c_0 \in T$  and  $t \hat{\ } c_1 \in T$ . For an  $\omega$ -tree  $T$ , the *set of branches of  $T$*  is  $[T] = \{f : \forall h (f \restriction h \in T) \wedge f \text{ is of maximal length}\}$ . For  $s \in {}^{\omega>}n$ , we have the basic open sets  $[s] = \{f \in {}^\omega n : f \supset s\}$  and the restricted trees  $T_s = \{t \in T : t \supseteq s\}$ . A tree  $T \subseteq {}^{\omega>}n$  is called  *$k$ -branching* if for all  $h < \text{ht}(T)$  and for all  $s \in {}^h n$  there are at least  $k$   $\ell$ 's such that  $s \hat{\ } \ell \in T$ . (So also all branches have the same length.)

$\forall^\infty$  means "for almost all", i. e. for all but finitely many. If  $X$  is a subset of the domain of a function  $f$ , then  $f''(X)$  denotes  $\{f(y) : y \in X\}$ .

**PROOF.** We assume that there were  $\alpha, \beta$  contradicting Theorem 1 and work toward a contradiction.

In the first step of the proof, we search for large subdomains of  ${}^\omega n \times [\omega]^\omega$  on which  $(\alpha, \beta)$  behave similarly to a connection between the "sharp" relations  $\mathbf{R}_n^\sharp$  and  $\mathbf{R}_m^\sharp$ . Thereafter we shall mimick the proof of [3, Theorem 1.4] for suitable restrictions of  $(\alpha, \beta)$  and thus get a contradiction. For  $k \in \omega$  we consider the sets

$$(1) \quad U_k = \{(f, X) \in {}^\omega n \times [\omega]^\omega : \alpha(f) \restriction X \text{ is not almost constant or } f \restriction (\beta(X) \setminus k) \text{ is constant}\}.$$

<sup>3)</sup>Added in proof: In summer 2001 OTMAR SPINAS obtained a stronger version of Theorem 1.

By our assumption on  $(\alpha, \beta)$ , we have that  $\bigcup_{k \in \omega} U_k = {}^\omega n \times [\omega]^\omega$ . We take the product measure  $\mu$  on  ${}^\omega n$  and the Lebesgue  $\lambda$  measure on the interval  $[0, 1]$  of the reals. Then we embed  $[\omega]^\omega$  into  $[0, 1]$  via characteristic functions  $\chi$ . The range of  $\chi$  has measure one but is not closed, and the measure on the range induces a measure on  $[\omega]^\omega$ . So we have  $(\mu \times \lambda)({}^\omega n \times [\omega]^\omega) = 1$ . By our assumptions on  $\alpha$  and  $\beta$  all the  $U_k$  are measurable with respect to  $\mu \times \lambda$ . Hence there is some  $k$  such that  $(\mu \times \lambda)(U_k) > 0$ . Since the measure  $\mu \times \lambda$  is approximated from below by measures of compact sets (see [4]), there is a compact set  $A_k \subseteq U_k$  such that  $(\mu \times \lambda)(A_k) = x > 0$ . We fix such an  $A_k$ , and we shall apply the Lebesgue Density Theorem [4, 3.20] to  $A_k =: A$ . For this purpose, we work with the balls  $B_h(f, X) = \{(g, Y) : (g, Y) \upharpoonright h = (f, X) \upharpoonright h\}$  of depth  $h$  around  $(f, X)$  for  $(f, X) \in {}^\omega n \times [\omega]^\omega$ , which have measure  $\frac{1}{n^h \cdot 2^h}$ . If  $A$  is measurable with respect to  $\mu \times \lambda$  and  $(\mu \times \lambda)(A) > 0$ , then we denote by  $\Phi(A)$  the subset of  $A$  of points at which  $A$  has Lebesgue density 1, i. e.,

$$\Phi(A) = \{(f, X) \in A : \lim_{h \rightarrow \infty} \frac{(\mu \times \lambda)(B_h(f, X) \cap A)}{n^{-h} \cdot 2^{-h}} = 1\}.$$

The Lebesgue Density Theorem says that  $(\mu \times \lambda)(A \Delta \Phi(A)) = 0$ .

Now we apply it  $\omega$  times successively in the following way: We take closed set  $B^{(0)} = A \cap \Phi(A)$  and  $B^{(n+1)} = B^{(n)} \cap \Phi(B^{(n)})$ . Now we set  $B = \bigcap_{\ell \in \omega} B^{(\ell)}$  and have that  $(\mu \times \lambda)(B) > x/2$  and that

$$\{(f, X) \in B : \lim_{h \rightarrow \infty} \frac{(\mu \times \lambda)(B_h(f, X) \cap B)}{n^{-h} \cdot 2^{-h}} = 1\} = B.$$

We fix some point  $(f, X) \in B$ . We take  $\varepsilon = 1/(2n)$  and get some  $h_0 \in \omega$  such that for all  $h \geq h_0$ ,  $\frac{(\mu \times \lambda)(B_h(f, X) \cap B)}{2^{-h} \cdot n^{-h}} > 1 - \varepsilon$ . This means that there are  $f_{i,j} \in B$ ,  $i < n$ ,  $j = 0, 1$ , such that  $f_{i,j} \supset (f, X) \upharpoonright h_0 \hat{\ } (h_0 + 1, (i, j))$ . Our next goal is to find countably many suitable trees  $T^y$ ,  $y \in {}^{>\omega}\omega$ , such that  $[T^y] \subseteq U_k$  and such that  $T^y$  be fully branching on certain levels.

For  $y = (y_0, \dots, y_{\ell-1}) \in {}^{>\omega}\omega$  we say that  $y$  is *sufficiently increasing* iff  $y_0 \geq h_0$  and for each  $i < \ell$ , if  $T^y \upharpoonright y_i$  is chosen and  $(f, X) \upharpoonright y_i \in T^y \upharpoonright y_i$ , and  $h$  is chosen minimally for  $\varepsilon = 1/(2n)$  and all these  $(f, X)$ 's as in the definition for being a member of  $B$ , then  $y_{i+1} \geq h$ .

*Claim.* *There is an infinite  $X$  such that for every  $y \in {}^{>\omega}\omega$ , if  $y$  is sufficiently increasing, then there are a tree  $T^y$  and some height  $r$ , such that  $T^y$  is fully branching on each level in  $\text{ran}(y) \cup (X \setminus r)$ , such that for all  $(g, X') \in [T^y]$ ,  $(f, X') \in U_k$ .*

*Proof of the Claim.* Let  $y \in {}^{>\omega}\omega$  be given. We have  $h_0 \in y_0$ , because otherwise  $y_0$  is not sufficiently increasing. By the definition of  $B$  we may replace  $h_0$  by any larger element. Now we take  $2n$  elements  $(f_{(i)}, X_{(i)}) \supset (f, X) \upharpoonright h_0$  such that  $(f_{(i)}, X_{(i)}) \upharpoonright h_0 + 1$  are all different. For  $i < 2n$  find  $h_1 > h_0$  such that for all  $h \geq h_1$ ,  $\frac{(\mu \times \lambda)(B_h(f_{(i)}, X_{(i)}) \cap B)}{2^{-h} \cdot n^{-h}} > 1 - \varepsilon$ . Similarly define  $(f_s, X_s)$  for  $s \in {}^{>(2n)}\omega$  by induction on  $|s|$ . Set  $T^y = \{(f_s, X_s) \upharpoonright h_{|s|}(y) : s \in {}^{>(2n)}\omega\}$ . Moreover, we choose the  $h_i(y)$ ,  $i \in \omega$ , above  $y_{\ell-1}$  in such a careful way, that there is an infinite pseudo-intersection  $X$  to  $\{\{h_i(y) : i \in \omega\} : y \in {}^{>\omega}\omega\}$ . Thus the Claim is proved.  $\square$

Now we work, until the end of Lemma 2, first on each

$$\pi_1(T^y) = \{f \upharpoonright h : (f, X) \upharpoonright h \in T^y\}, \quad y \in {}^{>\omega}\omega,$$

separately. We set  $H(y) = \{h_i(y) : i \in \omega\}$ . In the second step, we restrict the domain of  $\alpha$  to a comeager subset of  $\pi_1([T^y])$ . For this, we use the following lemma of KURATOWSKI [2]:

*Lemma 1. Let  $\mathcal{X}$  and  $\mathcal{Y}$  be topological spaces, and  $\mathcal{Y}$  have a countable base. For every Baire measurable function  $f : \mathcal{X} \rightarrow \mathcal{Y}$  there is a comeager set  $C \subseteq \mathcal{X}$  such that  $f \upharpoonright C$  is continuous (w. r. t. the subspace topology on  $C$  induced by  $\mathcal{X}$ ).*

We apply this lemma separately for each  $y$  to  $\mathcal{X} = \pi_1([T^y])$  and to  $\alpha$ , and thereafter thin out and homogenize further. In the following, until the end of Lemma 2, everything but  $X$  and  $X^*$  (see below) depends on  $y$  as well. However, in order to avoid too clumsy notation we do not index all objects all the time with the superscript  $y$ .

So, by the lemma we get a comeager  $C \subseteq \pi_1([T^y])$  such that  $\alpha \upharpoonright C$  is continuous. We take nowhere dense  $H_i$ ,  $i \in \omega$ , such that  $\pi_1([T^y]) \setminus C = \bigcup_{i \in \omega} H_i$ .

We now choose by induction on  $i$  an increasing sequence

$$n_0 = 0 < m_0 < n_1 < \dots < n_i < m_i < n_{i+1} < m_{i+1} < \dots$$

and  $s_i : [n_i, m_i) \rightarrow n$ ,  $t_i : [m_i, n_{i+1}) \rightarrow n$ .

Suppose  $n_i$  is chosen. Then we take  $m_i > n_i$  and  $s_i : [n_i, m_i) \rightarrow n$  such that  $H_i \cap \{f \in \mathcal{X} : f \upharpoonright [n_i, m_i) = s_i\} = \emptyset$ . (The existence of such  $m_i$ ,  $s_i$  is proved in [5, Théorème 21].) Now we take  $n_{i+1} > m_i$  and  $t_i : [m_i, n_{i+1}) \rightarrow n$  such that  $H_i \cap \{f \in \mathcal{X} : f \upharpoonright [m_i, n_{i+1}) = t_i\} = \emptyset$ . This ends the induction.

We define

$$\begin{aligned} T_0^y &= \{s \in \pi_1(T^y) : \forall i (m_i \leq \text{lh}(s) \rightarrow s \upharpoonright [n_i, m_i) = s_i)\}, \\ T_1^y &= \{s \in \pi_1(T^y) : \forall i (n_{i+1} \leq \text{lh}(s) \rightarrow s \upharpoonright [m_i, n_{i+1}) = t_i)\}. \end{aligned}$$

Now the square brackets also indicate the limit of the trees in the space  ${}^\omega n$ . Since  $[T_j^y] \cap \bigcup_{i \in \omega} H_i = \emptyset$ , the  $\alpha \upharpoonright [T_j^y]$  and hence  $\alpha \upharpoonright ([T_0^y] \cup [T_1^y])$  are continuous.

For later use, it is important that the  $T_j^y$  are fully branching on two sets of levels that combine to  $H(y)$ :

$$\begin{aligned} (\forall k \in \bigcup_{i \in \omega} [n_i, m_i) \cap H(y)) (\forall t \in T_1^y \upharpoonright k) (\forall j < n) (t \frown j \in T_1^y \upharpoonright (k+1)), \\ (\forall k \in \bigcup_{i \in \omega} [m_i, n_{i+1}) \cap H(y)) (\forall t \in T_0^y \upharpoonright k) (\forall j < n) (t \frown j \in T_0^y \upharpoonright (k+1)). \end{aligned}$$

Now let  $n > m$ . We are looking for an  $X'$  such that  $\beta(X')$  cannot be defined if it has to respect the morphism property. Such an  $X'$  will be found among the subsets of  $X$ . The following is sufficient for this aim:

$$(2) \quad \begin{aligned} (\exists X' \in [X]^\omega) ((\forall \beta(X') = Y' \in [\omega]^\omega) (\exists y \subset Y' \setminus k) \\ (\exists f \in \pi_1([T^y])) (\alpha(f) \upharpoonright X' \text{ is almost constant and} \\ f \upharpoonright ((\beta(X') \setminus k) \cap \text{ran}(y)) \text{ is not constant})). \end{aligned}$$

*Lemma 2. There is  $X^* \in [X]^\omega$  with the following property: For every sufficiently increasing  $y \in {}^{>\omega}\omega$ , there are  $S_j^y \subset T_j^y$  and  $g_j^y : X^y \rightarrow m$  such that  $S_1^y$  is branching on each level in  $\bigcup_{i \in \omega} [n_i^y, m_i^y) \cap H(y)$ ,  $S_0^y$  is branching in each level in  $\bigcup_{i \in \omega} [m_i^y, n_{i+1}^y) \cap H(y)$  and for all  $f \in [S_j^y]$ ,  $\alpha(f) \upharpoonright X^y = g_j^y$ , and  $X^* \setminus X^y$  is finite.*

*Proof.* First we choose for  $v \in \omega$ ,  $k_v^y$  such that for all  $v \in \omega$  and for all  $f \in [T_j^y]$ ,  $\alpha(f)(v)$  is determined by  $f \upharpoonright k_v^y$ . Such  $k_v^y$  exists by the continuity of  $\alpha \upharpoonright ([T_0^y] \cup [T_1^y])$ .

Now we select for every  $y$  successively a subset  $M_j^y$  of  $m$  that appears densely often at all the levels in  $X^y \in [X]^\omega$  in  $\alpha''((T_j^y))$  such that  $X^y$  is a subset of  $X^{y'}$  for  $y'$  being the successor of  $y$  in some enumeration of all the  $y$ 's of order type  $\omega$ : there are a subsequence  $\langle k_{u_v}^y : v \in \omega \rangle$  and for  $j = 0, 1$ ,  $M_j^y \subseteq m$  and  $r_j^y \in T_j^y \upharpoonright k_{u_0}^y$  such that for all  $t \in (T_j^y)_{r_j^y} \upharpoonright k_{u_v}^y$ ,  $g''((T_j^y)_t \upharpoonright k_{u_{v+1}}^y) = M_j^y$ , and  $\{u_v^y : v \in \omega\} \subseteq \{u_v^{y'} : v \in \omega\}$ . This follows from

$$(\forall W \in [\omega]^\omega)(\exists V \in [W]^\omega)(\exists M^y \subseteq m)(\exists r_j^y \in T_j^y)(\forall t \in (T_j^y)_{r_j^y})(\forall^\infty v \in V) \\ g''((T_j^y)_t \upharpoonright v) = M^y.$$

We set  $X^y = \{u_v^y : v \in \omega\}$  and choose for  $X^*$  such that it is a pseudointersection of  $X^y$ ,  $y \in {}^\omega > \omega$ .

Independently for each  $j$ , by induction on  $v \in \omega$ , we construct  $S_j^y \upharpoonright k_{u_v}^y$ . We begin with  $S_j^y \upharpoonright k_{u_0}^y = \emptyset$ . Suppose  $S_j^y \upharpoonright k_{u_v}^y$  is constructed, we take an arbitrary  $r$  out of this set. We select a good ‘‘color’’  $g_j^y(u_{v+1}) \in m$  such that more than  $1/\text{card}(M_j^y)$  points  $t$  in  $(T_j^y)_{r^y} \upharpoonright k_{u_{v+1}}^y$  and at least one point out of each  $(T_j^y)_{r'} \upharpoonright k_{u_{v+1}}^y$ ,  $r' \in S_j^y \upharpoonright k_{u_v}^y$ , fulfill  $g(t) = g_j^y(u_{v+1})$ . We put these points into  $S_j^y \upharpoonright k_{u_{v+1}}^y$ .

Every level of  $(S_0^y)_{r^y} \upharpoonright k_{u_{v+1}}^y$  in  $[k_{u_v}^y, k_{u_{v+1}}^y) \cap \bigcup_{i \in \omega} [m_i^y, n_{i+1}^y) \cap H(y)$  has a branching point, as otherwise in  $(S_0^y)_{r^y} \upharpoonright k_{u_{v+1}}^y$  there would be at most  $1/n$  times the maximal number  $n^z$ ,  $z = \text{card}([k_{u_v}^y, k_{u_{v+1}}^y) \cap \bigcup_{i \in \omega} [m_i^y, n_{i+1}^y))$ , of points. Similar for  $S_1^y$ .  $\square$

In order to finish the proof of Theorem 1, we take an infinite  $X' \subseteq X^*$  such that all  $(g_0^y, g_1^y) \upharpoonright X'$  are almost constant (and, of course, almost everywhere defined). This can be done because there are only countably many of them. Then for all  $y$  for all  $f \in [S_0^y] \cup [S_1^y]$  the restriction  $\alpha(f) \upharpoonright X'$  is almost constant, but there is no infinite  $\beta(X')$ , even no set with more than 3 elements  $\geq \max(k)$  in some suitable  $y$ , such that all  $f \in [S_0^y] \cup [S_1^y]$  are constant on  $\beta(X') \setminus k$ : Suppose that we had 3 such elements, say two of them,  $h_0$  and  $h_1$ , are in (the range of)  $y$  and in the branching points of  $S_0^{y_0}$  above  $k$ . Then we take  $f_0$  and  $f_1 \in [S_0^y]$  such that  $f_0 \upharpoonright (h_0 + 1) = f_1 \upharpoonright (h_0 + 1)$  and  $f_0(h_1) \neq f_1(h_1)$ . Then  $f_0 \upharpoonright \{h_0, h_1\}$  is not constant or  $f_1 \upharpoonright \{h_0, h_1\}$  is not constant. Thus Theorem 1 is proved.  $\square$

**References**

[1] BLASS, A., Reductions between cardinal characteristic of the continuum. In: Set Theory: Annual Boise Extravaganza in Set Theory Conference 1992-94 (T. BARTOSZYŃSKI and M. SCHEEPERS, eds.), Contemporary Mathematics Vol. 192, Amer. Math. Soc., Providence 1996, pp. 31 – 49.

[2] KURATOWSKI, K., Topology. Vol. 1. Academic Press, New York 1966.

[3] MILDENBERGER, H., Non-constructive Galois-Tukey connections. J. Symbolic Logic **62** (1997), 1179 – 1187.

[4] Oxtoby, J., Measure and Category. Second Edition. Springer-Verlag, Berlin-Heidelberg-New York 1980.

[5] TALAGRAND, M., Compacts de fonctions mesurables et filtres non mesurables. Studia Mathematicae **67** (1980), 13 – 43.

[6] VOJTÁS, P., Generalized Galois-Tukey Connections between explicit relations on classical objects of real analysis. In: Set Theory of Reals (H. JUDAH, ed.), Israel Math. Conference Proceeding Vol. 9, Amer. Math. Soc., Providence 1993, pp. 619 – 643.

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