FORCING WITH F_σ- AND WITH SUMMABLE FILTERS

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Abstract. We investigate forcing with summable filters and with F_σ-filters and forcing with infinite sets of natural numbers from the point of view whether their regular open algebras are isomorphic. We show that it is consistent with ZFC that these three notions of forcing are isomorphic. In the opposite direction, we show that the natural embedding of summable filters into F_σ-filters is not a complete embedding.

1. Introduction

Often forcings look very similar, and their generic filters share some distinguished properties, e.g., generating a P-point with no rapid ultrafilters Rudin-Keisler below it. We look at three notions of forcing, two of them adding such a special object:

The first is the forcing $\mathbf{P}_1 = (c_0 \setminus \ell^1, \leq^*)$, where c_0 is the set of real sequences that converge to 0, and ℓ^1 is the set of absolutely summable sequences. The binary relation $\leq^*$ is the partial order of eventual domination. The stronger condition is the $\leq^*$-smaller sequence which is still not summable.

The second forcing, $\mathbf{P}_2$, has as its domain all F_σ-filters on ω, and a stronger condition is a superset of a weaker one.

We take the forcing $\mathbf{P}_3 = ([\omega]^\omega, \subseteq^*)$ as a third object of study, because its comparison with the summable filter forcing brought up techniques that we shall be using, among others, also for the investigation of the first two forcings. Our main results are:

Theorem 1.1. *If* $\mathrm{cf}(2^\omega) = \mathfrak{p}$*, then the three forcings are equivalent.*

This extends a result of Vojtáš [10], which says that under the same premise the first an the third forcing are equivalent.

It is open whether this is a result in ZFC. Our trials to build isomorphisms failed right at the beginning:

2000 Mathematics Subject Classification 03E05, 03E35, 06E05, 40A05.

Theorem 1.2. *The map* $f \mapsto \{\omega \setminus X \mid \sum_{i \in X} f(i) < \infty\}$ *is not a complete embedding of* $\mathbf{P}_1$ *into* $\mathbf{P}_2$*.*

This should be compared to the much greater knowledge about $\mathbf{P}_1$ and $\mathbf{P}_3$:

Theorem 1.3. [3] *It is consistent that there is no complete embedding from* $\mathbf{P}_1$ *into* $\mathbf{P}_3$*.*

The forcing P_2 is not as large as it looks:

Theorem 1.4. [7, 6.3] *The forcing* $\mathbf{P}_2$ *is equivalent to forcing with filters generated by closed (in the Cantor space topology) sets.*

In [5] Just and Krawczyk investigate isomorphy types of Boolean algebras of the form $\mathscr{P}(\omega)/I$ for three concrete ideals I, one of them F_σ, the other two $F_{\sigma\delta}$. We will also look at quotients of the form $\mathscr{S}/J$ where $\mathscr{S}$ is a subset of $\mathscr{P}(\omega)$ or a small subset of $\mathscr{P}(\mathscr{P}(\omega))$ (that can be coded in $\mathscr{P}(\omega)$), and our ideals J on ω or on $\mathscr{P}(\omega)$ are given by the fact, that the forcings as written above are not separative and that we need to look at the separative quotient for determining forcing equivalence. The denominator J_3 for the forcing $\mathbf{P}_3$ is the ideal Fin of finite subsets of ω, and the other J are described via the relations $\approx_i$ given at the end of this section. In our scenario the F_σ-filters (or their dual ideals) are the conditions in the numerator of the quotient, in contrast to the mentioned work by Just and Krawczyk.

The three forcings are very similar in their generics: From any generic of filter G of $\mathbf{P}_1$ we can define

$$U_G = \{X \subseteq \omega \mid (\exists f \in G) \sum_{n \in \omega \setminus X} f(n) < \infty\}.$$

Laflamme [7, Section 6] showed that forcing with $\mathbf{P}_2$ adds a P-point with no rapid ultrafilter below it in the Rudin-Keisler ordering. Laflamme's proof works analogously for the forcing $\mathbf{P}_1$ and gives the same result for the ultrafilter U_G for generics G of $\mathbf{P}_1$. The forcing $\mathbf{P}_3$ adds a Ramsey ultrafilter by [8]. Although Ramsey ultrafilters are rapid, the difference in the generic objects does not necessarily imply that the notions of forcing are different. Our second result is a partial negative answer to the following question:

Question 1.5. *Is there a "definable" complete embedding of the forcing* $(c_0 \setminus \ell^1, \leq^*)$ *in the forcing* $(F_\sigma$*-filters*$, \supseteq)$*?*

Why do we speak about summable ultrafilters? Vojtáš [10] showed that the forcing $(c_0 \setminus \ell^1, \leq^*)$ is equivalent to forcing with summable filters. A filter F on ω is called summable, if there is a sequence $\langle a_i \mid i < \omega\rangle$ of positive reals such that $\sum_i a_i = \infty$ and $F = F_{\langle a_i \mid i<\omega\rangle} := \{\omega \setminus X \mid \sum_{i\in X} a_i < \infty\}$. Since the filters $F_{\langle a_i \mid i<\omega\rangle}$ are F_σ, the forcing with F_σ-filters is at least as a partial order a superstructure of the forcing with summable filters.

In the remainder of the section we review the relevant definitions and some facts. We write $A \subseteq^* X$ iff $A \setminus X$ is finite. A set $A \in [\omega]^\omega$ is called a pseudointersection of $\mathscr{X}$ if $(\forall X \in \mathscr{X})(A \subseteq^* X)$. The pseudointersection number $\mathfrak{p}$ is defined as $\min\{|\mathscr{X}| \mid \mathscr{X} \subseteq [\omega]^\omega$ is closed under finite intersections and there is no pseudointersection for $\mathscr{X}\}$.

We equip 2^ω with the Cantor space topology, i.e., a base for the open sets is $\{[s] \mid s \in 2^{<\omega}\}$ where $[s] = \{f \in 2^\omega \mid s \subseteq f\}$. A subset of $\mathscr{P}(\omega)$ is F_σ, if the set of the characteristic functions of its members is an F_σ-subset in 2^ω.

A filter on ω is a subset of $[\omega]^\omega$ that is closed under supersets and finite intersections. An F_σ-filter is a filter on ω such that the set of the characteristic functions of the sets in the filter is an F_σ-set.

An ultrafilter U on ω is a P-point if every countable subset $\{A_i \mid i < \omega\} \subseteq U$ has a pseudointersection $X \in U$. An ultrafilter U on ω is rapid, if for every $f \colon \omega \to \omega$ there is some $X \in U$ such that $(\forall n)|X \cap f(n)| \leq n$. For two ultrafilters U, V on ω we say U is below V is the Rudin-Keisler ordering, and we write $U \leq_{RK} V$, if there is some $f \colon \omega \to \omega$ such that $U = f(V) := \{X \subseteq \omega \mid f^{-1}X \in V\}$.

We recall what "equivalence of notions of forcing" means, and we recall the notion of embeddability. For our positive result we shall use the regular Boolean algebras of the forcings and criteria when complete Boolean algebras are isomorphic. For the negative result, it is more efficient to work with the partial orders directly.

A partial order $\mathbf{P} = (P, \leq)$ is a set P equipped with a reflexive and transitive relation $\leq$. When considering partial orders as notions of forcing, we take the convention that the smaller condition is the *stronger* one. We write $r \perp q$ iff $\neg\exists s\,(s \leq r \ \wedge\ s \leq q)$.

Definition 1.6. [6, 7.1] *Let $\mathbf{P} = (P, \leq_P)$ and $\mathbf{Q} = (Q, \leq_Q)$ be partial orders and $i \colon P \to Q$. The map i is a complete embedding iff*

(1) $(\forall p, p' \in P)(p' \leq_P p \to i(p') \leq_Q i(p))$, *and*

(2) $(\forall p, p' \in P)(p' \perp_P p \leftrightarrow i(p') \perp_Q i(p))$, *and*

(3) $(\forall q \in Q)(\exists p \in P)(\forall p' \in P)(p' \leq_P p \rightarrow (i(p') \not\perp_Q q)$. *Such a* p *is called a reduction of* q.

Definition 1.7. [6, 7.7] *A dense embedding* $i \colon P \rightarrow Q$ *is a complete embedding such that in addition* $i''P$ *is dense in* Q*, i.e.,* $(\forall q \in Q)(\exists r \in i''P)(r \leq_Q q)$.

Every generic extension $V[G]$ by a $\mathbf{Q}$-generic filter G over V contains (as a subset) a generic extension $V[i^{-1}G]$ by $\mathbf{P}$ iff there is a complete embedding $i \colon P \rightarrow Q$. For details, and proofs see [6, Chapter 7]. In terms of Boolean algebras, this is equivalent to having an embedding of the regular open algebras that preserves arbitrary meets and unions.

Two partial orders generate the same generic extensions (where G has to range over all generic filters) iff there is a dense embedding from one into the other, or equivalently, if their regular open algebras are isomorphic [6, Chapter 7].

A partial order $\mathbf{P} = (P, \leq)$ is separative, iff

$$\forall p, q \in P \, (p \not\leq q \longrightarrow \exists r \in P \; (r \leq p \; \wedge \; r \perp q)) \, ,$$

or, in topological terms, for $p \neq q \in P$ we have that $\mathrm{int}(\mathrm{cl}(\{p' \,|\, p' \leq p\})) \neq \mathrm{int}(\mathrm{cl}(\{q' \,|\, q' \leq q\}))$, where the interiors and closures are taken in the so-called cut topology on $(P, \leq)$, which is generated by the basic open sets $\{\{p' \,|\, p' \leq p\} \,|\, p \in P\}$. Hence the map $e_{\mathbf{P}} \colon p \mapsto \mathrm{int}(\mathrm{cl}(\{p' \,|\, p' \leqslant p\}))$ is a dense embedding into the Boolean algebra (minus its zero element) of regular open subsets of P, called $\mathrm{RO}(\mathbf{P})$.

In general, for a partial order $(P, \leq)$, $A \subseteq P$ is called regular open iff $\mathrm{int}(\mathrm{cl}(A)) = A$. As shown in [4, page 152], for any separative $(P, \leq)$ there is a unique complete Boolean algebra $\mathrm{RO}(\mathbf{P})$ into which — leaving out the Boolean algebra's zero element, of course —$\mathbf{P}$ is densely embedded. The same holds for not necessarily separative partial orders, only then the embedding is possibly not injective. Note that Definition 1.6 does not require that i be injective.

If a partial order $(P, \leq)$ is not separative, we may take the separative quotient (see [4, page 154]): We set $p \approx q$ iff $\forall r (r \perp p \leftrightarrow r \perp q)$. We denote the $\approx$-equivalence class of p by $p/\!\approx$ and the set of all equivalence classes by $P/\!\approx$. There is a partial order $\leq /\!\approx$ on the equivalence classes, defined by

$$p/\!\approx \;\; \leq /\!\approx \;\;\; q/\!\approx$$

iff

$$\forall r \in P(r \not\perp p \to r \not\perp q).$$

This is well defined, and $(P/{\approx}, \leq/{\approx})$ is separative. Note that neither of the $\mathbf{P}_i$ is separative. We write $\approx_i$ for the $\approx$-relation on $\mathbf{P}_i$. It is well-known that $\mathbf{P}_3/\approx_3 = ([\omega]^\omega/\mathrm{Fin}, \subseteq^*)$, and Vojtáš [10, 11] investigated the incomparability relation in $\mathbf{P}_1/{\approx_1}$. Let us call $\mathbf{B}_i = \mathrm{RO}(\mathbf{P}_i/{\approx_i})$.

Now we can state the technical versions of the questions we are interested in: Are there complete (or even dense) embeddings from $\mathbf{P}_i/{\approx_i}$ to $\mathbf{P}_j/{\approx_j}$?

2. All three can be the same

In this section we proof Theorem 1.1. The techniques Vojtáš developed and used in his work on $\mathbf{P}_1$ and on $\mathbf{P}_3$ yield a proof, once it is shown that the sufficient conditions are fulfilled by $\mathbf{P}_2$ as they are fulfilled by $\mathbf{P}_1$.

We use Balcar and Simon's chapter on "Disjoint refinement" in the Handbook of Boolean Algebras [1], in order to show that if $\mathfrak{p} = \mathrm{cf}(2^\omega)$ then $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{B}_3$ are isomorphic. The proof follows the outline given in [10], therefore we only sketch it here. We shall use the following for $\kappa = 2$, $\lambda = \mathfrak{p}$, $\tau = \mathrm{cf}(2^\omega)$, $\rho < \tau$.

Definition 2.1. *(1) A Boolean algebra $\mathbf{B}$ is (ρ, κ)-distributive iff for each set $\{P_\alpha \mid \alpha < \rho\}$ of maximal antichains P_α there is a maximal antichain Q such that for each $\alpha < \rho$ and each $q \in Q$ we have that*

$$|\{p \in P_\alpha \mid p \cdot_{\mathbf{B}} q \neq 0\}| < \kappa.$$

For $\kappa = 2$ this amounts to ordinary ρ-distributivity.

A Boolean algebra $\mathbf{B}$ is (ρ, κ)-nowhere distributive iff for each $x \in B \setminus \{0\}$ the algebra $\mathbf{B} \restriction x$ is not (ρ, κ)-distributive.

(2) A subset C of the domain of a Boolean algebra $\mathbf{B}$ is λ-closed iff for all $\leq_{\mathbf{B}}$-descending sequences $\langle c_i \mid i < \gamma\rangle$ from C, $\gamma < \lambda$, there is some $c \in C$ such that for all $i < \gamma$, $c \leq_{\mathbf{B}} c_i$.

(3) A subset C of the domain B of a Boolean algebra $\mathbf{B}$ is a π-base iff for every $b \in B$ there is some $c \in C$ such that $c \leq_{\mathbf{B}} b$.

Theorem 2.2. *[Balcar, Simon]* [1, Theorem 1.13] *If $\tau, \lambda \geq \aleph_0$, $\kappa \geq 2$ and B is a (τ, κ)-nowhere distributive Boolean algebra having a λ-closed dense subset C and being $(\rho, 2)$-distributive for every $\rho < \tau$ and having a π-base of cardinality $\kappa^{<\lambda}$,* *then there is a dense subset $T \subseteq C$ of B such that $(T, \geq)$ is a tree of height τ such that each $t \in T$ has $\kappa^{<\lambda}$ successors.*

Now we apply Theorem 2.2 with $\lambda = \mathfrak{p}$ ($\mathbf{B}_i$ are $\mathfrak{p}$-closed), $\kappa = 2$ and $\tau = \mathrm{cf}(2^\omega)$ (the $\mathbf{B}_i$ are not $\mathrm{cf}(2^\omega)$-distributive, as shown in [10]). However, the $\mathbf{B}_i$ are ρ-distributive for every $\rho < \tau = \mathfrak{p}$, since they are $\mathfrak{p}$-closed. Now, all the premises of Theorem 2.2 are fulfilled, and it gives the same tree for all three $\mathbf{B}_i$'s. $\Box_{1.1}$

Remark 2.3. Take $i \neq j$. Then an isomorphism from $\mathbf{B}_i$ to $\mathbf{B}_j$, call it e, is possibly inconstructive in the following sense: We are aiming at a lifting ψ_e such that the outer parts of the Diagram 1 would be commutative. The first step is: There are densely many $p \in P$ such that $e(\{p' \mid p' \leq p\})$ has the form $\{q' \mid q' \leq q\}$. For these p we set $\varphi_e(p) = q$. Then φ_e is defined on a dense subset, say $D/\approx_i$ of $\mathbf{P}_i/\approx_i$, and it is a complete embedding from $(D, \leq_{P_i})/\approx_i$ to $\mathbf{P}_j/\approx_j$. It is open whether there is a Borel-lifting ψ_e, such that the diagram is commutative:

$$\begin{array}{ccc}
(D, \leq_{\mathbf{P}_i} \restriction (D \times D)) & \xrightarrow[\psi_e]{} & \mathbf{P}_j \\
\Big\downarrow \pi & & \Big\downarrow \pi \\
(D, \leq_{\mathbf{P}_i} \restriction (D \times D))/\approx_i & \xrightarrow[\varphi_e]{} & \mathbf{P}_j/\approx_j \\
\Big\downarrow e_{\mathbf{P}_i/\approx_i} & & \Big\downarrow e_{\mathbf{P}_j/\approx_j} \\
\mathbf{B}_i & \xrightarrow[e]{} & \mathbf{B}_j
\end{array}$$

Diagram 1

On liftings of Boolean algebras (regarded as structures with finitary Boolean operations, not as complete algebras in the forcing sense) and in particular on liftings of maps from $\mathscr{P}(\omega)/I$ and $\mathscr{P}(\omega)/J$ for ideals I, J, the reader may consult the survey article [2] and the references given there.

In the forcing case, the question on the kind of lifting is analogous and multifaceted: Suppose that the P_i are subsets of 2^ω or ω^ω or $[\omega]^\omega$, such that being Borel is defined. Then one can ask whether some Borel ψ_e exists on a suitable domain D. The domain D may be replaced by any $\leq_{P_i}$-dense set of representatives of $\mathbf{P}_i/\approx_i$. Moreover, there are various natural choices to describe $\approx_i$-classes in $\mathbf{P}_i$, and they are often Borel computable from each other. In contrast to this, Theorem 2.2 gives inconstructive dense embeddings, which exist in some but possibly not in all models of ZFC.

3. Embeddings

In [3] it is shown that $\mathbf{B}_1$ and $\mathbf{B}_3$ can be not isomorphic. Motivated by this result we work towards showing differences between the notions of forcing. If they are isomorphic in some ZFC-models but are not isomorphic in others this shows that the isomorphisms existing only in some models cannot be absolute. So we try to exclude some possible candidates for isomorphisms who just come from a Borel lifting ψ (in Diagram 1).

Definition 3.1. *The mapping id "identity" is the following*

$$\begin{aligned}&\mathrm{id}\colon P_1/{\approx_1} \to P_2/{\approx_2},\\ &f/{\approx_1} \mapsto (\{\omega \setminus X \mid \sum_{n\in X} f(n) < \infty\})/{\approx_2} = F_f/{\approx_2}.\end{aligned}$$

It is well defined by Vojtáš [11].

Now we prove Theorem 1.2. We use results of Mazur [9]: There are F_σ-filters that are not contained in any summable filter. This idea, that there are many more F_σ-filters than summable filters is used to build an F_σ-filter I^* that does not have a reduction (as in Definition 1.6 (3)) in the summable filters.

We take an arbitrary unbounded $h\colon \omega \to \mathbb{N}$ and iteratively choose finite intervals K_n in ω (whose lengths are written as $|K_n|$) such that

$$K_n = \left[\sum_{i<n} |K_i|, \sum_{i<n} |K_i| + (n \cdot h(n))^n\right); \tag{3.1}$$

Then we take a function $k\colon \omega \to \mathbb{R}_+$ such that

$$\sum_{n\in\omega} k(n) = \infty; \tag{3.2}$$

$$\sum_{n\in\omega} \frac{k(n)}{h(n)} < \infty; \tag{3.3}$$

and finally we choose a function $f\colon \omega \to \mathbb{R}_+$ such that

$$\sum_{i\in K_n} f(i) = k(n). \tag{3.4}$$

For example, we may choose $h(n) = n$ and $k(n) = n^{-\alpha}$ for some $0 < \alpha < 1$ and any f such that equation (3.4) holds.

Let $f/{\approx_1}$ be the equivalence class of f in the separative quotient of P_1. We show that there is some $I^* \leq_{\mathbf{P}_2} F_f$ that is $\mathbf{P}_2$-incompatible with any summable filter below $\mathrm{id}(f/{\approx_1})$. This shows that $I^*/{\approx_2}$ does not have any reduction, and hence the embedding is not complete.

Lemma 3.2. *(*[9, Lemma 1.8]*) For any $n \in \omega \setminus \{0\}$ and $\varepsilon \in \mathbb{R}_+$ there is a set K_n and a family $\mathcal{R}_n$ of subsets of K_n such that*

a) $\forall v_1, \dots, v_n \in \mathcal{R}_n \; v_1 \cap \dots \cap v_n \neq \emptyset$.
b) *If P is any probability distribution on K_n then there is some $v \in \mathcal{R}_n$ such that $P(v) < \varepsilon$.*
c) $|K_n| = \left(\frac{n}{\varepsilon}\right)^n$. □

We set $\mathcal{R}_n^* = \{K_n \setminus v \,|\, v \in \mathcal{R}_n\}$.

Theorem 3.3. *(*[9, Theorem 1.9]*) For all sequences $\langle \varepsilon_n \,|\, n \in \omega \rangle$ of positive reals there is some F_σ-ideal I and there is a partition $\langle K_n \,|\, n \in \omega \rangle$ of ω into finite sets K_n such that on $\mathcal{P}(K_n)$ there are $\mathcal{R}_n$ as in the previous lemma and*

(1) $|K_n| = \left(\frac{n}{\varepsilon_n}\right)^n$.
(2) *$I =$ the ideal generated in $\mathcal{P}(\omega)$ by $\{u \,|\, (\forall^\infty n \in \omega)\; u \cap K_n \in \mathcal{R}_n^*\}$.*
(3) *If P_n is any probability distribution on K_n then there is some $u \in \mathcal{R}_n^*$ such that $P(u) \geq 1 - \varepsilon_n$.* □

Mazur shows that such an I is not contained in any summable ideal. Our aim is to strengthen this by showing that under an appropriate choice of ε_n we have that I^* is $\mathbf{P}_2$-stronger than $\mathrm{id}(f/{\approx_1})$ and $\mathbf{P}_2$-incompatible with any summable filter below $\mathrm{id}(f/{\approx_1})$.

Proof: We fix f, h, k and $\langle K_n \,|\, n \in \omega \rangle$ as in equations (3.1) to (3.4). We take $\varepsilon_n = \frac{1}{h(n)}$. Then we have for the ideal I constructed according to Lemma 3.2 and Theorem 3.3 and the probability distributions P_n on K_n with

$$P_n(\{i\}) = \frac{f(i)}{\sum_{i \in K_n} f(i)}$$

that

$$\forall n \; \exists u_n \in \mathcal{R}_n^* \sum_{i \in u_n} f(i) \geq \left(1 - \frac{1}{h(n)}\right) \sum_{i \in K_n} f(i).$$

By definition of I, we have that $u = \bigcup_{n \in \omega} u_n \in I$. Now we have for every $f' \leq^* f$, say $f'(n) \leq f(n)$ for $n \geq k$, that $\omega \setminus u \in I_{f'} := \{X \subseteq \omega \,|\, \sum_{n \in X} f'(n) < \infty\}$, because

$$\sum_{n\in\omega}\sum_{i\in K_n\setminus u} f'(i) \leq \sum_{n\in\omega}\sum_{i\in K_n\setminus u} f(i) + \sum_{i\leq k} f'(i) \leq \sum_{n\in\omega}\frac{1}{h(n)}\sum_{i\in K_n} f(i) + \sum_{i\leq k} f'(i) < \infty.$$

Hence $I^* = \{\omega\setminus X \mid X \in I\}$ is $\mathbf{P}_2$-incompatible with any summable filter $F_{f'} = I^*_{f'}$ for $f' \leq^* f$. $\square_{1.2}$

Acknowledgement: We thank Andreas Blass for having us asked whether the first two notions of forcing are equivalent.

References

[1] Bohuslav Balcar and Petr Simon. Disjoint refinement. In J. Donald Monk and R. Bonnet, editors, *Handbook of Boolean Algebras, vol. II*, pages 35–67. North-Holland, Amsterdam, 1989.

[2] Ilijas Farah. Completely additive liftings. *Bull. Symbolic Logic*, 4:37–54, 1998.

[3] Sakaé Fuchino, Heike Mildenberger, Saharon Shelah, and Peter Vojtáš. On absolutely divergent series. *Fund. Math.*, 160:255–268, 1999.

[4] Thomas Jech. *Set Theory.* Addison Wesley, 1978.

[5] Winfried Just and Adam Krawczyk. On Certain Boolean Algebras $\mathscr{P}(\omega)/I$. *Trans. Amer. Math. Soc.*, 285:411–429, 1984.

[6] Kenneth Kunen. *Set Theory, An Introduction to Independence Proofs.* North-Holland, 1980.

[7] Claude Laflamme. Forcing with filters and complete combinatorics. *Ann. Pure Appl. Logic*, 42:125–163, 1989.

[8] Adrian Mathias. Happy families. *Ann, Math. Logic*, 12:59–111, 1977.

[9] Krzysztof Mazur. F_σ-ideals and ω_1, ω_1^*-gaps in $\mathcal{P}(\omega)/I$. *Fund. Math.*, 138:103–111, 1991.

[10] Peter Vojtáš. Boolean isomorphism between partial orderings of convergent and divergent series and infinite subsets of N. *Proc. Amer. Math. Soc.*, 117:235 – 242, 1993.

[11] Peter Vojtáš. On ω^* and absolutely divergent series. *Top. Proceedings*, 19:335 – 348, 1994.

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