TREES, STATIONARY REFLECTION, AND MAHLO CARDINALS

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A major thread of set-theoretic research focuses on realizing the compactness properties of large cardinals at accessible cardinals like $\aleph_2$ or $\aleph_{\omega+1}$, thus answering questions that one could naturally pose without realizing that large cardinals are relevant. We discuss recent work with Thomas Gilton and Šárka Stejskalová in which we realized an array of consistency results concerning variants of the tree property and the stationary reflection—two frequently-studied types of compactness properties—for double successors of regular cardinals like $\aleph_2$. These kinds of results are usually obtained by starting with an embedding $j : M \to N$ that witnesses the given type of large cardinal κ, and then by reasoning in a forcing extension $M[\mathbb{P}]$ that collapses κ while preserving the properties of interest.

This talk will focus on two results from our paper that involve Mahlo cardinals, which are challenging to work with because the associated embeddings take the form $j : M \to N$ where N is closer to the model in which we hope to realize the result; hence the domain of the embedding contains less information. The first result to be discussed follows a theorem of Gilton and Krueger that showed that the consistency of a Mahlo cardinal is sufficient to produce a model in which the so-called approachability property fails for κ^{++} and in which every stationary subset of $\kappa^{++} \cap \mathrm{cof}(\leq \kappa)$ reflects. We produced a new proof using more conventional argumentation, but which works because of a characterization of $\Diamond_\lambda(\mathrm{Reg})$ that we introduced for Mahlo cardinals. The other result we discuss has to do with the fact that compactness properties sometimes determine the value of the continuum. On the contrary, we produced a model in which every stationary subset of $\kappa^{++} \cap \mathrm{cof}(\leq \kappa)$ reflects, there are no κ^{++}-special Aronszajn trees, and 2^κ is as large as we like.

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