

Lie Groups
SoSe 2020 — Übungsblatt 1
Ausgabe 11.05.20, Abgabe 19.05.19

Solutions are due on Tuesday 19th May at 23:59. Please send it by email at

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Group work is encouraged!

1 Topological groups

Aufgabe 1.1: Show that $(\mathbb{R}^\times, \cdot)$ is a topological group (with the standard subspace topology on \mathbb{R}^\times).

(4 Punkte)

Aufgabe 1.2: Let G be a topological group and let H be a subgroup of G .

1. Show that H is open if and only if it contains a neighborhood of the identity $e \in G$.
2. Show that if H is open, then H is also closed.
3. Show that if G is connected and H is open then $G = H$.

(4 Punkte)

Bonus-Aufgabe 1.3: Let $m, n \in \mathbb{Z}$ with $m \neq 0$. We define $d_5(m, n) = \frac{1}{5^k}$ where k is the maximal integer such that 5^k divides $m - n$. If $m = n$ let $d_5(m, n) = 0$. Then d_5 defines a metric on \mathbb{Z} called the 5-adic metric (one can do the same for every prime p).

Show that $(\mathbb{Z}, +)$ is a topological group with respect to the topology induced by the metric d_5 .

(4 Punkte)

2 Representation theory of S^1

Aufgabe 1.4: For $n, m \in \mathbb{Z}$ let $\rho_{n,m} : S^1 \times S^1 \rightarrow GL_1(\mathbb{C}) \cong \mathbb{C}^\times$ be the map defined by $\rho_{m,n}(z, z') = z^m (z')^n$.

1. Show that $\rho_{m,n}$ defines a representation of $S^1 \times S^1$.

2. Show that $\rho_{m,n}$ is isomorphic to $\rho_{m',n'}$ (as a representation) if and only if $m = m'$ and $n = n'$.
3. **Bonus:** Show that every representation of $S^1 \times S^1$ on \mathbb{C} is isomorphic to $\rho_{n,m}$ for some $n, m \in \mathbb{Z}$.

(4 + 2 Punkte)