Lie Groups
SoSe 2020 — Ubungsblatt 7

Ausgabe 10.07.20, **Abgabe** 23.07.20

Solutions are due on Tuesday 28thd July at 23:59. Please send it by email at

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Aufgabe 7.1: Show that $SO_3(\mathbb{R})$ has a maximal abelian group isomorphic to $(\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z})$.
(In particular, this exercises shows that not every maximal abelian subgroup in a compact Lie group is a torus.)

(4 Punkte)

Aufgabe 7.2: Let K be a compact Lie group. Show that the center of K is the intersection of all the maximal tori in K.

(4 Punkte)

Aufgabe 7.3: Let

$$T = \left\{ \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \mid \theta \in \mathbb{R} \right\}$$

be a maximal torus of $SO_3(\mathbb{R})$. Show that the root system $R(SO_3(\mathbb{R}), T) = \{1, -1\}$, where 1 denotes a generator of $\mathcal{X}(T) \cong \mathbb{Z}$.
Hint: compute the action of T in the basis $\{E_1 + iE_2, E_1 - iE_2, E_3\}$ of $\mathfrak{so}_{\mathbb{R}}(3) \otimes \mathbb{C}$.

(4 Punkte)

Aufgabe 7.4: Let X be a lattice and $X^\vee := \mathrm{Hom}_k(X, \mathbb{Z})$ be the dual lattice. Let s be a reflection in X and let $\alpha \in X$ be a corresponding root with coroot $\alpha^\vee \in X^\vee$.
Show that the map $s^\vee : X^\vee \to X^\vee$ defined by $s^\vee(\lambda)(v) = \lambda(s(v))$ for any $\lambda \in X^\vee$, $v \in X$ is a reflection in $X^\vee$ with $\alpha^\vee$ as root and α as coroot.

(4 Punkte)

The 12th and last lecture will be uploaded on July 31st. There will be no exercises for that lecture.