Lie Groups SoSe 2023 — Ubungsblatt 9 12.07.2023

Aufgabe 9.1: Let G be a Lie group and $T \subset G$ be a maximal torus. Let $f: G \to H$ be a surjective homomorphism of Lie groups and suppose H is abelian. Show that the restriction of G to T is also surjective.

Aufgabe 9.2: Let G be a compact connected Lie group. Show that the center of G is the intersection of all its maximal tori

Aufgabe 9.3: Let G be a compact connected Lie group and $g \in G$. Let $Z(g)^{\circ}$ be the connected component of the identity of the centralizer of g. Show that $Z(g)^{\circ}$ is the union of the maximal tori in G containing g. Show that the centralizer of $g = \begin{pmatrix} 1 \\ -1 \\ & -1 \end{pmatrix} \in SO(3, \mathbb{R})$ is not connected.

Aufgabe 9.4: Compute the Weyl group of $SO(3, \mathbb{R})$.