Nichtkommutative Algebra und Symmetrie
SS 2019 — Übungsblatt 8

22. Juni 2019

Informationen zur Vorlesung finden Sie unter:
`http://home.mathematik.uni-freiburg.de/soergel/ss19nkas.html`

Exercise 8.1: Show that $\exp : M_2(\mathbb{R}) \to GL_2(\mathbb{R})$ is neither surjective nor injective.

Exercise 8.2: Let $\langle -, - \rangle$ be the standard scalar product on $\mathbb{R}^{2n}$ and let

$$J = \begin{pmatrix} 0 & Id_n \\ -Id_n & 0 \end{pmatrix} \in M_{2n}(\mathbb{R}).$$

The standard symplectic form ω is defined as $\omega(v, w) = \langle v, Jw \rangle$.
The symplectic group $Sp_{2n}(\mathbb{R})$ is the Lie group of matrices preserving the standard symplectic form ω, i.e.

$$Sp_{2n}(\mathbb{R}) = \{A \in GL_n(\mathbb{R}) \mid A^\top JA = J\}$$

Show that $T_I Sp_{2n}(\mathbb{R}) = \{M \in M_{2n}(\mathbb{R}) \mid M^\top - JMJ = 0\}$ and compute $\dim(Sp_{2n}(\mathbb{R}))$.

Exercise 8.3: Let $U \subset GL_n(\mathbb{R})$ be the Lie subgroup of upper triangular matrices with only 1 on the diagonal. Show that

$$exp : T_I U \to U$$

is a diffeomorphism.
Hint: Consider the logarithm.

Exercise 8.4: Let G be a commutative Lie subgroup of $GL_n(\mathbb{R})$.

- Show that $exp : T_I G \to G$ is a group homomorphism.
- Assume the G is connected. Then show that $exp : T_I G \to G$ is surjective.
- Show that $G \cong \mathbb{R}^m/\Gamma$, where Γ is a discrete subgroup of $\mathbb{R}^m$.