Corrections for W. Soergel: On the relation between intersection cohomology and representation theory in positive characteristic, pp. 311-335 in Journal of Pure and Applied Algebra 152 (2000)

1. The argument on page 329 at the bottom needs fixing. Thanks are due to Masaharu Kaneda for pointing this out and insisting. Let us reprove Proposition 3.2.6. First we treat the case that $\mathcal{F} = i_{n!}\overline{B/B}$ is the skyscraper at the one-point cell. In this case the claim follows from the fact that $i_{n!}i_n^!\mathcal{E} \to \mathcal{E}$ induces an injection on hypercohomology, which follows from the degeneration of the spectral sequence computing $\mathbb{H}^\bullet\mathcal{E}$ explained in the proof of 3.2.6. Then, for the general case, it will be sufficient to check the commutativity of the following diagrams, for $\mathcal{A} \in \mathcal{D}(G/P_s)$ and $\mathcal{F} \in \mathcal{D}(G/B)$:

$$
\begin{array}{ccc}
\mathrm{Hom}_\mathcal{D}(\pi^*\mathcal{A}, \mathcal{F}) & \longrightarrow & \mathrm{Hom}(\mathbb{H}^\bullet\pi^*\mathcal{A}, \mathbb{H}^\bullet\mathcal{F}) \\
\downarrow{\scriptstyle\wr} & & \downarrow \\
\mathrm{Hom}_\mathcal{D}(\mathcal{A}, \pi_*\mathcal{F}) & \longrightarrow & \mathrm{Hom}(\mathbb{H}^\bullet\mathcal{A}, \mathbb{H}^\bullet\pi_*\mathcal{F})
\end{array}
$$

for the map on the right coming from $\mathcal{A} \to \pi_*\pi^*\mathcal{A}$, and

$$
\begin{array}{ccc}
\mathrm{Hom}_\mathcal{D}(\mathcal{F}, \pi^!\mathcal{A}) & \longrightarrow & \mathrm{Hom}_C(\mathbb{H}^\bullet\mathcal{F}, \mathbb{H}^\bullet\pi^!\mathcal{A}) \\
\downarrow{\scriptstyle\wr} & & \downarrow{\scriptstyle\wr} \\
\mathrm{Hom}_\mathcal{D}(\pi_!\mathcal{F}, \mathcal{A}) & \longrightarrow & \mathrm{Hom}_{C^s}(\mathbb{H}^\bullet\pi_!\mathcal{F}, \mathbb{H}^\bullet\mathcal{A})
\end{array}
$$

Here the point is to construct dually to Proposition 4.1.1 a canonical isomorphism $\mathrm{Hom}_{C^s}(C, \mathbb{H}^\bullet\mathcal{A}) \xrightarrow{\sim} \mathbb{H}^\bullet\pi^!\mathcal{A}$ and show that the resulting diagram will commute. With these diagrams, a non-injective case in 3.2.6 would lead to a noninjective case with $\mathcal{F}$ the skyscraper, which we have already shown to be impossible.

2. I add some details concerning the proof of Corollary 1.0.3. First, it is important to be aware of 3.3.4 and the following lines to follow the argument, as these lines say, that pulling back a non-semisimple perverse sheaf from a partial flag variety to the full flag variety and shiftig the degree to obtain a perverse sheaf again, the perverse sheaf obtained will be non-semisimple as well. The point is now that, if Lusztig's conjecture is ok, then the projectives of the modular $\mathcal{O}$ have the same Verma flag multiplicities as they do for the classical non-modular $\mathcal{O}$, given by KL-polynomials for the finite Weyl group. However, if the decomposition theorem would not be ok at some step of the inductive construction of these special indecomposable

complexes now called parity sheaves, and we take the first such step, these projectives would turn out bigger for the very parameter this step of their inductive construction leads to.