ÄQUIVARIANTE η-INVARIANTEN HOMOGENER RÄUME

English Summary

In our Ph-D thesis, we calculate the G-equivariant η-invariant of a family of generalized Dirac operators on a quotient $M = G/H$ of compact connected Lie groups. More precisely: If $M = G/H$ is G-spin, let ΣM be the G-equivariant spinor bundle of M, let $U^\kappa M$ be the homogeneous vector bundle induced by the irreducible unitary H-representation with heighest weight κ and let D_M^κ be the natural Dirac operator on $\Sigma M \otimes U^\kappa M$. We construct a "reductive" Dirac operator $\tilde{D}_M^\kappa$ which agrees with D_M^κ up to zero-order terms.

Now let $T \supset S$ be maximal tori of G and H and suppose that G/T is G-spin as well. Then one can reduce the computation of $\eta_G(\tilde{D}_M^\kappa)$ to the torus T/S:

$$\eta_G\bigl(\tilde{D}_M^\kappa\bigr)(g,s) = \eta_G\bigl(\tilde{D}_{G/S}^{\kappa+\rho_H}\bigr)(g,s) = \frac{A_G\bigl(\eta_T(\tilde{D}_{T/S}^{\kappa+\rho_H})(\,\cdot\,,s)\bigr)}{A_G(\rho_G)}\,(g)$$

for $g \in T$ and $s \in \mathbb{C}$; here ρ_H and ρ_G are half sums of positive roots, and A_G denotes the alternating sum over the Weyl group of G. By computing $\eta_T(\tilde{D}_{T/S}^{\kappa+\rho_H})$ one gets an explicit formula for $\eta_G(\tilde{D}_M^\kappa)$ which holds with slight modifications if G/T or M are not G-spin. Furthermore, $\eta_G(\tilde{D}_M^\kappa)$ vanishes identically if $\operatorname{rk} G - \operatorname{rk} H$ is larger than 1.

The function $\eta_G(\tilde{D}_M^\kappa)$ "approximates" $\eta_G(D_M^\kappa)$ (in a non-technical sense): Restricted to the subset G_0 of G that acts freely on M, $\eta_G(\tilde{D}_M^\kappa) - \eta_G(D_M^\kappa)$ is a virtual character of G, which vanishes if D_M is the untwisted Dirac operator on M for a normal metric. Thus the formula from above can be applied to compute some $\mathbb{R}/\mathbb{Z}$-valued invariants of flat bundles over locally homogeneous spaces of the type $\Gamma\backslash G/H$.

If M is a compact symmetric space we have $\tilde{D}_M^\kappa = D_M^\kappa$. We calculate the equivariant η-invariants of the untwisted Dirac operator and of the tangential boundary operator of the signature complex for all irreducible symmetric spaces with $\operatorname{rk} G - \operatorname{rk} H = 1$ (i.e., for the odd dimensional real Grassmannians and for $\mathrm{SU}_3/\mathrm{SO}_3$).

As a function on G, the equivariant η-function $\eta_G(D)$ can have singularities at all elements $g \in G$ that don't act freely. If $M = \partial N$ for some manifold N and if D and the G-action can be extended to N in a compatible way, then these singularities can be described by an integral over a certain equivariant differential form on the fixed point set of g on M. One can thus recover $\eta_G(D_M)$ on all of G if one only knows its values on G_0. We use this method together with our formula for $\eta_G(\tilde{D}_M)$ to calculate $\eta_G(D_M)$ for the Aloff-Wallach spaces $M = \mathrm{SU}_3/S^1$.

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