Spectral convergence in geometric quantization

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In this series of lectures I explain geometric quantization from the viewpoint of spectral convergence. We take a Lagrangian fibration on a symplectic manifold and a family of compatible complex structures tending to the real polarization given by the fibration, and show a spectral convergence of the d-bar Laplacian on the prequantum line bundle to the spectral structure related to the set of Bohr–Sommerfeld fibers. This talk is based on the joint work with Mayuko Yamashita.

Lectures

1. **Bohr-Sommerfeld fibers and the convergence of holomorphic sections.**
   I explain the foundations of Kähler polarization, real polarization and the relation between the holomorphic sections and the eigenfunctions on some Riemannian manifolds.

2. **Spectral structures and the measured Gromov–Hausdorff convergence.**
   I define the convergence of a sequence of metric measure spaces and explain Cheeger–Colding's results which shows the convergence of the spectral structure of Laplacians, under the assumptions that the lower bound of the Ricci curvatures are given. I also explain the notion of the convergence of the spectral structures introduced by Kuwae–Shioya

3. **Convergence of the spectral structures of d-bar Laplacians on the prequantum bundle.**
   I explain the main theorem of this course. We consider the convergence of the spectral structures of d-bar Laplacians on the prequantum bundle in the case of abelian manifolds, toric symplectic manifolds and K3 surfaces.

4. **Convergence of the frame bundle of the prequantum bundles.**
   To show the main theorem, we explain the convergence of the frame bundle of the prequantum bundles equipped with the Riemannian metrics related to the hermitian connection and the complex structures.