$$\mathcal{L}_{X_t}\omega_t = -\beta$$ → Moser's equation

$$\omega_1 - \omega_0 = d\beta \Longleftrightarrow [\omega_0] = [\omega_1]$$

(Moser)

$$[\omega_1] = [\omega_0] \Longleftrightarrow \omega_1 - \omega_0 = d\beta_\omega$$

$$\omega_t = (1-t)\,\omega_0 + t\,\omega_1$$

$$d\omega_t = 0$$

$$\mathcal{L}_{X_t}\omega_t = -\beta$$

X_t: unknown; ω_t: obtc; β: obtc

$$X_t(\varphi_t) = \left.\frac{d\varphi_s}{ds}\right|_{s=t}$$

Some formula (look at notes for the proof)

$$\frac{d}{dt}(\varphi_t^*\omega_t) = \varphi_t^*\left(\frac{d\omega_t}{dt} + \mathcal{L}_{X_t}\omega_t\right)$$

$d\omega_t$ …

$$\frac{\dot{}}{dt} = \frac{d}{dt}\left((1-t)w_0 + t w_1\right) = w_1 - w_0 = +d\beta$$

$$\mathcal{L}_{X_t} w_t = d\,\iota_{X_t} w_t + \iota_{X_t} d w_t = d\,\iota_{X_t} w_t = +d\beta$$

$$\mathcal{L}_X w = d\,\iota_X w + \iota_X dw$$ Cartan's magic formula

vioton

$$\iota_{X_t} w_t = -\beta$$ Moser's equation

time-t-flow

$$\Rightarrow \frac{d}{dt}(\varphi_t^* w_t) = 0$$

$t = 0 \quad \varphi_0 = Id$

$$\varphi_t^* w_t = w_0$$

well-defined

$$d(t_i, t_j) = w(X_{t_i}, X_{x_j})$$ ← Definition of Poisson bracket

X_{t_i}

$$\iota_{X_{t_i}} w = -dt_i$$

$\{t_i, t_j\} = X_{t_i}(dt_j)$ $\qquad f = \mathcal{F}$

$0 = \{t_i, t_j\} = X_{t_i}(dt_j)$

$\Longrightarrow$ X_{t_i} are tangent to the fibers

$F = (t_1, \dots, t_n)$

$\mathcal{D} = \langle X_{t_1}, \dots, X_{t_n} \rangle$ $\longrightarrow$ $\mathcal{F}$ = fibration

$[X_{t_i}, X_{t_j}] = X_{\{t_i, t_j\}} = 0$

exercise

$\phi : \mathbb{R}^n \times \mathcal{L}(c) \longrightarrow \mathcal{L}(c)$

$((s_1 \dots s_n), x) \longrightarrow (\phi^{s_1}_{X_{t_1}} \circ \phi^{s_2}_{X_{t_2}} \circ \dots (x))$

(Duistermaat's trick)

ϕ defines a group action

$L \to$ compact. $\Leftarrow$ $L \simeq G/G_x = \mathbb{R}^n / G_x$

$\to L$ $\quad G_x \to$ lattice of $\mathbb{R}^n$

$L = \mathbb{R}^n /$ $\quad \longrightarrow L = \mathbb{T}^n$

n_x

Liouville theorem

Thm (Liouville)

Compact levels of an integrable system are tori.

Uniformization of isotropy groups

$\to \quad \phi : \mathbb{T}^n \times \mathcal{L}(\mathbb{T}^n) \longrightarrow \mathcal{L}(\mathbb{T}^n)$

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