Lecture 1

Sunday, August 1, 2021 9:15 PM

homological methods

BV formalism in pAQFT

AQFT (algebraic quantum field theory)

↳ axiomatic approach to QFT

$\mathbb{M} = (\mathbb{R}^4, (^{1}{}_{-1}{}_{-1}{}_{-1}))$ Minkowski

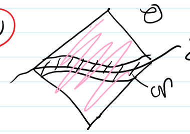
$\cup$

$\mathcal{O} \longmapsto \mathfrak{A}(\mathcal{O})$ algebra of observables that can be measured in $\mathcal{O}$

bounded

C^*-algebra, von Neumann algebra

Axioms:

- $\mathcal{O} \subset \mathcal{O}' \Rightarrow \mathfrak{A}(\mathcal{O}) \subset \mathfrak{A}(\mathcal{O}')$ isotony
- $\mathcal{O}_1 \times \mathcal{O}_2$ spacelike then $[\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = \{0\}$ causality
- 

  Σ Cauchy surface (every timelike curve intersects it exactly once)

  then $\mathfrak{A}(\mathcal{N}) \cong \mathfrak{A}(\mathcal{O})$ time-slice axiom

Haag "Local quantum physics"

Locally covariant QFT: generalization to globally hyperbolic spacetimes

↳ i.e. has a cauchy surface $M \cong \Sigma \times \mathbb{R}$

→ perturbative

pAQFT – $\mathfrak{A}(\mathcal{O})$ is a formal power series in $\hbar$ with values in topological, unital $*$-algebras.

→ differential graded

dgAQFT – $\mathfrak{A}(\mathcal{O})$ is a differential graded algebra

Def: Let R be a commutative ring. A chain complex $(C_\bullet, d)$ is a sequence of modules:

$$\dots \xleftarrow{d_0} C_0 \xleftarrow{d_1} C_1 \xleftarrow{d_2} C_2 \xleftarrow{d_3} \dots \quad \text{s.t. } d^2 = 0$$

(the cochain complex is dual to that, so the arrows go opposite direction)

Def: A differential graded algebra over R is either

- a chain complex $A_\bullet$ of R-modules endowed with R-bilinear maps: $A_n \times A_m \longrightarrow A_{n+m}$ such that:
  $(a, b) \longmapsto ab$

  $$d_{n+m}(ab) = (d_n a) b + (-1)^n a\, d_m b$$

  $\bigoplus A_n$ is associative and unital

or

•) a cochain complex (....)

<u>Def:</u> The elements $\mathrm{Ker}(d)$ – (co)cycles (closed)
$\mathrm{Im}(d)$ – (co)boundaries (exact)

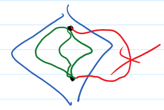
$$\underline{H_n} = \frac{\mathrm{Ker}\, d_n}{\mathrm{Im}\, d_{n+1}} \qquad \left(\underline{H^n} = \frac{\mathrm{Ker}\, d^n}{\mathrm{Im}\, d^{n-1}} \right)$$

<u>Example</u>: de Rham cohomology $(\bigoplus \Omega^n(M), d, \wedge)$
$\uparrow$ n-forms

M – spacetime: globally hyperbolic, time-oriented, oriented..

Caus(M) – collection of relatively compact, connected, contractible causally convex, open subsets $\mathcal{O} \subset M$

contains all causal curves with endpoints in $\mathcal{O}$



$\hookrightarrow$ objects
morphisms: inclusions $\Big\}$ category

<u>Def:</u> Let $PAlg^*$ the category of differential graded topological Poisson algebras with involution

<u>Def:</u> A dg classical field theory model on a spacetime M is a functor
$$\mathcal{P}: \mathrm{Caus}(M) \to PAlg^* \text{ that obeys:}$$

•) <u>Causality</u> holds on homology level $\left(\{ \mathcal{P}(\mathcal{O}_1), \mathcal{P}(\mathcal{O}_2) \} = \{0\} \text{ for } \mathcal{O}_1 \times \mathcal{O}_2 \right)$

•) <u>Time-slice</u> axiom holds on homology level

<u>Def:</u> Let $Alg^*_\hbar$ be the category of formal power series in $\hbar$ with coefficients in differential graded topological $*$-algebras

<u>Def:</u> A dg quantum theory model on M is a functor
$$\mathfrak{A}: \mathrm{Caus}(M) \to Alg^*_\hbar \text{ that obeys:}$$

•) <u>Causality</u> holds on homology level

•) <u>Time-slice</u> axiom holds on homology level

For more detail and relation to factorisation algebras, see:

Relating nets and factorization algebras of observables: free field theories

[Owen Gwilliam](), [Kasia Rejzner]()

From <https://arxiv.org/abs/1711.06674>