

$$\langle X, Y \rangle = \left\langle \frac{\delta X}{\delta \phi}, \frac{\delta Y}{\delta \phi^\dagger} \right\rangle - \left\langle \frac{\delta X}{\delta \phi^\dagger}, \frac{\delta Y}{\delta \phi} \right\rangle$$

Observation: $\delta X = -\int_{\text{dL}} X = -\langle dL, X \rangle = -\left\langle \frac{\delta L(\phi)}{\delta \phi(x)}, X \right\rangle$ $\phi \equiv 1$ on $\text{supp } X$

$$= \{X, L(\phi)\} \quad (\phi \equiv 1 \text{ on } \text{supp } X)$$

$$\equiv \{X, L\}$$

$$X = \int X(x) \frac{\delta}{\delta \phi(x)} \equiv \phi^\dagger(x) \text{ anti-field}$$

In the situation with symmetries (H^{-1} is non-trivial) we replace E with some graded vector space \bar{E} (extended configuration space)

of (anti)field number counts the number of ϕ^\dagger generators

Example: For Yang-Mills with gauge group G . Take $\mathfrak{g} \equiv \text{Lie } G$

and define $\bar{E} = E \oplus \mathfrak{g}[1]$

functions on \bar{E} \rightarrow odd generators called ghosts

$\mathcal{F}^{\text{multifield}}$ functions on \bar{E}

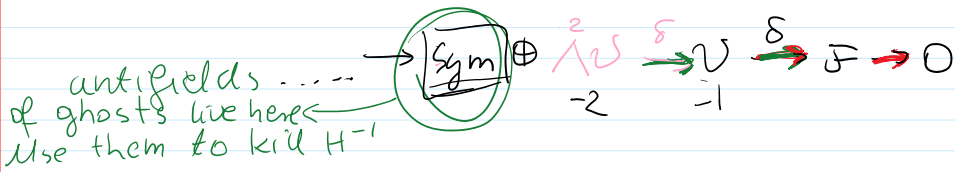
$$T^*[-1] \bar{E} = E \oplus \mathfrak{g}[1] \oplus E^*[-1] \oplus \mathfrak{g}^*[-2]$$

anti-fields

\hookrightarrow extend the lagrangian to L^{ext} , and hence the differential:

$$\mathfrak{s} \equiv \{., L^{\text{ext}}\} = \delta + \delta + \dots$$

anti-field #: $-1 \ 0 \ + \dots$



I want this complex to be a resolution

NB: We have an odd symplectic structure on $T^*[-1] \bar{E}$: " $\delta \phi \wedge \delta \phi^\dagger$ "

Relation to symplectic reduction (using BV-BFV):

Introduction to the BV-BFV formalism, [Alberto S. Cattaneo, Nima Moshayedi](https://arxiv.org/abs/1905.08047) <<https://arxiv.org/abs/1905.08047>>

NB: We can also write things in terms of a bi-complex:

- \rightarrow with δ horizontal \rightsquigarrow is a resolution
- \rightarrow with δ vertical
- $\rightarrow \mathfrak{s}$ is the total differential

Fad: $H^0(s, BV) = H^0(\delta, H^0(\delta, BV))$
 multilocal functionals $T^*[E] \bar{E}$
 going on-shell (taking functions on the solution space)
 taking the space of symmetric inv. functions

Output: \bar{E} - graded v. space (extended config. space)
 L^{ext} - extended Lagrangian $\rightsquigarrow s = \{., L^{ext}\}$
 $L_0 + V \rightarrow$ interaction
 $L_0 = L_{00} + \Theta_0$ quadratic part
 $\# \text{ of } \Theta_0 = 0$
 knows about linearized symmetries
 \rightarrow the rest

Assume that: $L_{00}^{(2)}$ induces a normally hyperbolic operator $P \rightsquigarrow$ constant Poisson bivector Π

\rightarrow get the Perels bracket:

$$[F, G] = \langle \Pi, dF \otimes dG \rangle$$

III Quantization of the free theory

free classical theory: $\mathcal{P}(\Theta) = (BV(\Theta), L, \cdot, \cdot, s_0 = \{., L_0\})$
 multilocal functionals on $T^*[E] \bar{E}$

Deformation quantization:

$$F \star G = \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \langle W^{\otimes n}, F^{(n)} \otimes G^{(n)} \rangle, \text{ where:}$$

$$W = \frac{i}{2} \Pi + H \rightarrow \text{symmetric part}$$

Choice of this symmetric part has to satisfy:

- 1) W is a bisolution for P ($\langle W, P f \otimes g \rangle = 0 = \langle W, f \otimes P g \rangle$)
- 2) W is positive definite ($\langle W, \bar{f} \otimes f \rangle \geq 0$)
- 3) Condition on the wavefront set of W (microlocal spectrum condition)
- 4) s_0 has to be a derivation w.r.t. \star

NB: from 1) it follows that $\{., L_0\}$ is a derivation w.r.t. \star

Non-trivial requirement: $\{., \Theta_0\}$ is a derivation as well (can be formulated as a condition for W)

References:

Fredenhagen, K., Rejzner, K.: Batalin-Vilkovisky formalism in perturbative algebraic quantum field theory. Commun. Math. Phys. **317**(3), 697–725 (2012)

Hollands, S.: Renormalized quantum Yang-Mills fields in curved spacetime. Rev. Math. Phys. **20**, 1033–1172 (2008). [arXiv:gr-qc/705.3340v3](https://arxiv.org/abs/gr-qc/705.3340v3)

Rejzner, K.: Remarks on local symmetry invariance in perturbative algebraic quantum field theory. Ann. Henri Poincaré 1–34 (2013)

Example: Linear observables: $\Phi(f) \in \mathcal{O} = \int \varphi(x) f(x)$

$$[\Phi(f), \Phi(g)]_{**} = i\hbar \underbrace{\langle \pi, f \otimes g \rangle}_{[\Phi(f), \Phi(g)]}$$