Lecture 4

Thursday, August 5, 2021 10:13 AM

$$\{X, Y\} = \left\langle \frac{\delta_r X}{\delta\varphi}, \frac{\delta_l Y}{\delta\varphi^\ddagger} \right\rangle - \left\langle \frac{\delta_r X}{\delta\varphi^\ddagger}, \frac{\delta_l Y}{\delta\varphi} \right\rangle$$

Observation: $\delta X = -\iota_{dL} X = -\langle dL, X\rangle = -\left\langle \frac{\delta L(f)}{\delta\varphi(x)}, X \right\rangle$ $f \equiv 1$ on supp X

$= \{X, L(f)\}$ $\underbrace{f \equiv 1 \text{ on supp } X}$

$\{X, L\}$ (|||)

$$X = \int X(x) \frac{\delta}{\delta\varphi(x)}$$ $\frac{\delta}{\delta\varphi(x)} \equiv \varphi^\ddagger(x)$ antifield

In the situation with symmetries (H^{-1} is non-trivial) we replace $\mathcal{E}$ with some graded vector space $\bar{\mathcal{E}}$ (extended configuration space)

$\#$ af (antifield number) counts the number of $\varphi^\ddagger$ generators

Example: For Yang-Mills with gauge group G. Take $\mathfrak{g} \equiv \mathrm{Lie}\, G$ and define $\bar{\mathcal{E}} = \mathcal{E} \oplus \mathfrak{g}[1]$

odd generators called ghosts

functions on $\bar{\mathcal{E}}$

$C^\infty(\mathcal{E}, \mathbb{C}) \hat{\otimes} \bigwedge^\bullet \mathfrak{g}^* \supset \mathcal{F}$ multilocal functions on $\bar{\mathcal{E}}$

$$T^*[-1]\bar{\mathcal{E}} = \mathcal{E} \oplus \mathfrak{g}[1] \oplus \underbrace{\mathcal{E}^*[-1] \oplus \mathfrak{g}^*[-2]}_{\text{antifields}}$$

I extend the lagrangian to L^{ext}, and hence the differential:

$$s \doteq \{\cdot, L^{ext}\} = \delta + \gamma + \dots$$

antifield #: -1 0 $+\dots$

$$\boxed{\mathrm{Sym}} \oplus \bigwedge^2 \mathcal{V} \xrightarrow{\delta} \underset{-2}{} \mathcal{V} \xrightarrow{\delta} \underset{-1}{} \mathcal{F} \to 0$$

antifields ..... → (Sym)

antifields of ghosts live here. Use them to kill H^{-1}

I want this complex to be a resolution

NB: We have an odd symplectic structure on $T^*[-1]\bar{\mathcal{E}}$: "$\delta\varphi \wedge \delta\varphi^\ddagger$"

Relation to symplectic reduction (using BV-BFV):

Introduction to the BV-BFV formalism, [Alberto S. Cattaneo](), [Nima Moshayedi]() <https://arxiv.org/abs/1905.08047>

NB: We can also write things in terms of a bi-complex:

- → with δ horizontal ⇝ is a resolution
- → with γ vertical
- → s is the total differential

Fact: $H^0(s, \mathcal{BV}) = H^0(\gamma, H^0(\delta, \mathcal{BV}))$ — taking the space of symmetry inv. functions

$H^0(s,\mathcal{BV})$ = multilocal functionals $T^*[-1]\overline{\mathcal{E}}$

$\underbrace{H^0(\delta, \mathcal{BV})}$ going on-shell (taking functionals on the solution space)

Output: $\overline{\mathcal{E}}$ – graded v. space (extended config. space)

L^{ext} – extended Lagrangian $\rightsquigarrow$ $s = \{\cdot, L^{ext}\}$

$L^{ext} = L_0 + V$ — V → interaction

L_0 ↳ quadratic part $L_0 = L_{00} + \Theta_0$

L_{00}: $\#af = 0$

Θ_0 ↳ the rest — knows about linearized symmetries

Assume that: $L_{00}^{(2)}$ induces a normally hyperbolic operator P $\rightsquigarrow$ constant Poisson bivector Π

I get the Peierls bracket:

$$\lfloor F, G \rfloor = \langle \Pi, dF \otimes dG \rangle$$

III Quantization of the free theory

free classical theory: $\mathfrak{P}(\mathcal{O}) = (\mathcal{BV}(\mathcal{O}), \lfloor \cdot, \cdot \rfloor, s_0 = \{\cdot, L_0\})$

$\mathcal{BV}(\mathcal{O})$ = multilocal functionals on $T^*[-1]\overline{\mathcal{E}}$

Deformation quantization:

$$F \star G = \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \langle W^{\otimes n}, F^{(n)} \otimes G^{(n)} \rangle, \text{ where:}$$

$$W = \frac{i}{2}\Pi + H$$ — H → symmetric part

Choice of this symmetric part has to satisfy:

1) W is a bisolution for P $(\langle W, Pf \otimes g\rangle = 0 = \langle W, f \otimes Pg \rangle)$
2) W is positive definite $(\langle W, \bar{f} \otimes f\rangle \geq 0)$
3) Condition on the wavefront set of W (microlocal spectrum condition)
4) s_0 has to be a derivation w.r.t. $\star$

NB: from 1) it follows that $\{\cdot, L_{00}\}$ is a derivation w.r.t. $\star$

Non-trivial requirement: $\{\cdot, \Theta_0\}$ is a derivation as well (can be formulated as a condition for W)

References:

Fredenhagen, K., Rejzner, K.: Batalin-Vilkovisky formalism in perturbative algebraic quantum field theory. Commun. Math. Phys. **317**(3), 697–725 (2012)

Hollands, S.: Renormalized quantum Yang-Mills fields in curved spacetime. Rev. Math. Phys. **20**, 1033–1172 (2008). arXiv:gr-qc/705.3340v3

Rejzner, K.: Remarks on local symmetry invariance in perturbative algebraic quantum field theory. Ann. Henri Poincaré 1–34 (2013)

Example: Linear observables: $\Phi(f)[\varphi] = \int \varphi(x) f(x)$

$$[\Phi(f), \Phi(g)]_\star = i\hbar \underbrace{\langle \Pi, f \otimes g \rangle}_{\lfloor \Phi(f), \Phi(g) \rfloor}$$