Übungsaufgaben zur Vorlesung
DG II | SS2025, Serie 02
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Aufgabe 1 (4 Punkte)
Assume $M = \mathbb{R} \times \mathbb{S}^{n-1}$ be a simply connected manifold with metric

$$g = dr^2 + \lambda^2(r) f_{ij}(\theta) d\theta^i d\theta^j,$$

where $f_{ij}(\theta)d\theta^i d\theta^j$ is the metric on the standard sphere $\mathbb{S}^{n-1}$. We have proved that if g has constant sectional curvature K, then

$$\Lambda(r) = \begin{cases} \frac{1}{\sqrt{K}} \sin(\sqrt{K}r), & \text{if } K > 0 \\ r, & \text{if } K = 0 \\ \frac{1}{\sqrt{-K}} \sinh(\sqrt{-K}r), & \text{if } K < 0 \end{cases}$$

Let $\partial B(r)$ denote the geodesic sphere centered at $r = 0$ of radius r, and denote by $\bar{J}(r)$ the area element of $\partial B(r)$. You compute $\bar{J}(r)$ and check that it satisfies

$$\bar{J}'' = \frac{n-2}{n-1}(\bar{J}')^2 \bar{J}^{-1} - (n-1)K\bar{J}$$

Aufgabe 2 (4 Punkte)

Let $\mathbb{S}^2$ be viewed as a plane $\mathbb{R}^2$ by the stereographic projection with the metric

$$g = \frac{4}{(1 + x^2 + y^2)^2}(dx^2 + dy^2).$$

Let $B_p(r)$ be a geodesic ball centered in p of radius r. Compute the volume of $B_p(r)$ and the area of $\partial B_p(r)$

Aufgabe 3 (4 Punkte)

Prove that a complete manifold (M^m, g) with the property that

$$Ric \geq 0, \quad \text{and} \quad \lim_{r\to\infty} \frac{vol(B_p(r))}{\omega_m r^n} = 1$$

for some $p \in M$, must be isometric to the Euclidean space.

Aufgabe 4 (4 Punkte)

Let N_t be a family of n-dimensional hypersurfaces in $\mathbb{R}^{n+1}$ with a normal variation vector field T and let H_t be the mean curvature of N_t. Compute the variational formula for H_t

$$\frac{d}{dt}H_t.$$

Hint. The computation is included in the computation of the second variational formula.

Bitte schreiben Sie Ihre(n) Namen, die Matrikelnummer sowie die Nummer Ihrer Übungsgruppe auf jedes Lösungsblatt. **Abgabe ist am Montag, 12.5.25, vor der Vorlesung**.