Übungsaufgaben zur Vorlesung
DG II — SS2025, Serie 07
Prof. Dr. G. Wang — 16.6.25
Dr. X. Zhang

Aufgabe 1 (4 Punkte)

Let Σ^n be a hypersurface in $\mathbb{R}^{n+1}$ satisfying

$$\langle x, N(x)\rangle = 0, \quad \forall x \in \Sigma,$$

where $N(x)$ is the unit normal of Σ at $x \in \Sigma$. Prove that Σ is conical, i.e, roughly speaking $t \cdot \Sigma = \Sigma$ for any t

Aufgabe 2 (4 Punkte)

Suppose $\Sigma^n \subset \mathbb{R}^{n+1}$ is a smooth minimal submanifold and $x_0 \in \Sigma$. Assume further that Σ is a graph of a function $u : \Omega \to \mathbb{R}$. Prove that the density defined by

$$\Theta_{x_0} := \lim_{s\to 0} \Theta_{x_0}(s)$$

is equal to 1. Here

$$\Theta_{x_0}(s) = \frac{Vol(B_s(x_0) \cap \Sigma)}{Vol(B_s^n \subset \mathbb{R}^n)}.$$

Aufgabe 3 (4 Punkte)

Prove that the catenoid, which is defined by

$$\{(\cos\theta \cosh t, \sin\theta \cosh t, t) \,|\, \theta \in (0, 2\pi), t \in \mathbb{R}\},$$

is a minimal surface.

Aufgabe 4 ($4 + 4^*$ Punkte)

Define a functional F on hypersurfaces Σ in $\mathbb{R}^{n+1}$ by

$$F(\Sigma) = \int_\Sigma \exp\left(-\frac{|x|^2}{4}\right)$$

1. Compute the first variation of F and the Euler-Lagrange equation for critical points of F (These critical points are called self-shrinkers and come up in mean curvature flow);
2. Compute the second variation of F for a compactly supported normal variation;

3. Show that there are no closed hypersurfaces that are stable critical points for F .

Bitte schreiben Sie Ihre(n) Namen, die Matrikelnummer sowie die Nummer Ihrer Übungsgruppe auf jedes Lösungsblatt. **Abgabe ist am Montag, 23.6.25, vor der Vorlesung**.