Problem 1 *(Open Balls)*
Let (X, d) be a metric space. Show that the ball

$$B(x, r) = \{y \in X : d(y, x) < r\} \quad \text{with } x \in X,\ r > 0,$$

is an open subset.

Problem 2 *(Continuous map)*
Recall that a function $f\colon (a, b) \to \mathbf{R}$ is called *strictly monotonic* if it is either strictly increasing (i.e., $x < y \Rightarrow f(x) < f(y)$ for all $x, y \in (a, b)$) or strictly decreasing (i.e., $x < y \Rightarrow f(x) > f(y)$ for all $x, y \in (a, b)$).
Using this definition, prove that any continuous injective function $f\colon (a, b) \to \mathbf{R}$ form an open interval to $\mathbf{R}$ must be strictly monotonic.

Problem 3 *(Homeomorphism)*
Let $f\colon (a, b) \to \mathbf{R}$ be a continuous injective function as in Problem 2. Prove that the image $f((a, b))$ is an open interval, and that the map

$$f\colon (a, b) \to f((a, b))$$

is a heomorphism.

Problem 4 *(The Cross)*
Using the results from previous problems, show that the set

$$K = \{(x, y) \in \mathbf{R}^2 : xy = 0\}$$

with the subspace topology is not a (toplogical) manifold.

Submit solutions by Tuesday, October 21st, before 6:00 PM to Ernst-Zermelo-Str. 1, mailbox on the 3rd floor, or directly to me during Tuesday's class.