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Parabolic Partial Differential Equations

SS 2019

Exercise Sheet 10

Turn in by Thursday, July 11.

1. *(2P)* Find the solution to the initial value problem

$$\begin{cases} \partial_t u - \Delta u = \cos(t) & \text{on } \mathbb{R}^n \times (0,\infty) \\ u(x,0) = \exp(-|x|^2) & \text{for } x \in \mathbb{R}^n \end{cases}$$

2. *(2P)* Given $u_0 \in C([0,2\pi])$ with $u(0) = u(2\pi)$, find a solution for the following initial value problem

$$\begin{cases} \partial_t u - D\Delta u = 0 & \text{on } [0,2\pi] \times (0,\infty) \\ u(x,0) = u_0(x) & \text{for } x \in [0,2\pi] \end{cases}$$

using the fundamental solution.

3. *(2P)* Let $\Omega \subset \mathbb{R}^n$ be an open subset with C^1-boundary. Given $u_0 \in C(\Omega)$ and $f \in C(\Omega \times [0,T])$, show that there is at most one $C^{2,1}$ solution to the following initial value problem with Neumann boundary conditions

$$\begin{cases} \partial_t u - \Delta u = f & \text{in } \Omega \times [0,T] \\ u(x,0) = g(x) & \text{for } x \in \Omega \\ \frac{\partial u}{\partial \nu}(x,t) = 0 & \text{for } x \in \partial\Omega \end{cases}$$

Hint: Integration by parts!

4. *(2P)* Describe three methods to find solutions of the heat equation in different settings. For each method, state the function spaces in which the initial and boundary conditions can be taken. What is the regularity of the resulting solutions?