The framework: threshold-linear networks

A non-empty subset of neurons $a_1, \ldots, a_p$ is a stable set of $(W, D)$ if there exist an asymptotically stable fixed point $x^* \in \mathbb{R}^n$ whose support is a non-degenerate square distance matrix $A$ is a stable set of $(W, D)$.

A perturbative approach:

$$M = -11^2 + \varepsilon A$$

where $\varepsilon > 0$ and $A$ is a Hermitian matrix.

A geometric observation:

$$M(M_1) = \frac{M_1 + M_2}{2} = \frac{M_1 + M_2 - M}{6}$$

This generalizes! Main geometric result:

Theorem. Let $\varepsilon > 0$ and $M$ an $n \times n$ Hermitian matrix for $n \geq 2$. Then the perturbed matrix $-11^2 + \varepsilon A$ is a stable matrix if and only if the following two conditions hold:

1. $A$ is a non-degenerate square distance matrix, and
2. $0 < \varepsilon < C(A)$, where $C(A)$ is the Cayley-Menger determinant of $A$.

This theorem allows us to construct symbolic matrices for many prescribed lists of memory patterns.

Problem: Find a 6x6 symmetric matrix whose stable principal submatrices are given by this simplicial complex.

Solution:

$$M(z) = \begin{pmatrix}
1 & z & z & z & z & z \\
1 & 1 & z & z & z & z \\
1 & 1 & 1 & z & z & z \\
1 & 1 & 1 & 1 & z & z \\
1 & 1 & 1 & 1 & 1 & z \\
1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}$$

The geometric result allows us to construct symbolic matrices for many prescribed lists of memory patterns.

Conclusions

1. Within this framework of memory encoding, we can encode sets of highly overlapping memory patterns exactly using a perturbative approach.
2. Precise encoding of overlapping memory patterns can be achieved using a geometric characterization for the stability of principal submatrices of the symbolic matrix.