# Exercise Sheet 1
## Algebraic Topology II

*Please write your name on your solution sheet. The submission deadline is thursday, 25th of April, 14h (post box "Jonas Schnitzer", 3rd floor, Ernst-Zermelo-Straße)*

**Exercise 1 (10 points)** Prove or disprove:

(i) Homology functors are cocontinuous.

(ii) $\mathbb{C}P^{n-1} \leftarrow \mathbb{C}P^n \leftarrow S^{2n+1}$ is a cofiber sequence.

(iii) Let $X$ be a topological space, such that all continuous maps $X \to \mathbb{Z}$ are constant, then $\pi_0(X) = 0$.

(iv) For every pair $(X, A)$ in $\mathcal{T}_+$ and every (reduced, generalised) homology functor $\tilde{h}$, the sequence

$$\tilde{h}_k(X/A) \longleftarrow \tilde{h}_k(X) \longleftarrow \tilde{h}_k(A)$$

is exact.

(v) A map between CW spectra $f\colon \mathbb{E} \to \mathbb{F}$ is invertible if it induces isomorphisms $\pi_k(\mathbb{E}) \to \pi_k(\mathbb{F})$ for all $k \in \mathbb{Z}$

**Exercise 2 (10 points)** Let $f\colon X \to Y$ be a weak equivalence of cofibrant spaces and $E$ be cofibrant (both in Quillen's model structure), then

$$f \wedge \mathrm{id}_E\colon X \wedge E \to Y \wedge E$$

is a weak equivalence of cofibrant spaces as well.

**Exercise 3 (10 points)** Let $R$ be a unital commutative ring. Check that the tensor product and the internal hom-functor fulfill the properties from Definition 4.26 and Bemerkung 4.28. This turns $(\mathcal{M}od_R, \otimes_R, R)$ into a closed monoidal category. What is its exponential law?

**Exercise 4 (10 points = 5+5 Points)** Let $X$, $Y$, $Z$ be pointed topological spaces and let $p\colon X \times Y \to X \wedge Y$, $q\colon (X \wedge Y) \times Z \to (X \wedge Y) \wedge Z$ and $r\colon X \times Y \times Z \to X \wedge Y \wedge Z$ be quotient maps. Prove the following:

(i) The identity map $X \wedge Y \wedge Z \to (X \wedge Y) \wedge Z$ is continuous in $\mathcal{T}op$.

(ii) The inverse is continuous in $k\mathcal{T}op$ or $kw\mathcal{H}$.

*Hint:* Use Proposition 4.32.