Exercise Sheet 2

Algebraic Topology II

Please write your name on your solution sheet. The submission deadline is thursday, 2nd of May, 14h (post box "Jonas Schnitzer", 3rd floor, Ernst-Zermelo-Straße)

Exercise 1 (10 points) Prove or disprove:

(i) Let $G = G_1 \times G_2$, then $K(G_1, p) \times K(G_2, p)$ is an Eilenberg-Mac Lane space for G.

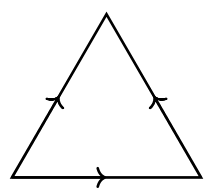
(ii) $\tilde{H}_8(K(\mathbb{Z}, 2)) \cong \mathbb{Z}$.

(iii) $\pi_4(\mathbb{C}P^2 \wedge \mathbb{C}P^3) \cong \mathbb{Z}$.

(iv) There exist a cofiber sequence $S^1 \to S^3 \to S^2$.

(v) Let G be an abelian group, let $S \subseteq G$ be a subgroup and let $Q = G/S$ be the quotient group. If X is a $K(G, n)$ and if Y is a $K(Q, n)$, then the natural map from Satz 5.43 is a fibration with fiber $K(S, n)$.

Exercise 2 (10 points = 4+4+2+0+0 points) The topological dunce hat X is constructed by identifying all edges of a triangle as in the figure below.



(i) Show that $\tilde{H}^{CW}_\bullet(X; \mathbb{Z}) = 0$.

(ii) Compute the fundamental group of X.

(iii) Show that X is contractible.

(iv) Indicate an explicit homotopy $h\colon X \times I \to X$ between the identity and a constant map.

(v) Make a model.

Exercise 3 (10 points) Let R be a ring Ring and let $\mathcal{M}od_R$ be the category of (right-) R-modules. Show Satz 5.44 for R, by understanding an R-modul M as an abelian group, and elements $r \in R$ as endomorphsims of M. Interpret $\tilde{H}^\bullet(\cdot\,; M)$ as an R-modul and prove (1)–(3).

Exercise 4 (10 points) Let the following be a commutative diagram of abelian groups with exact rows

$$\begin{array}{ccccccc} & & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle a} & & \downarrow{\scriptstyle b} & & \downarrow{\scriptstyle c} & & \\ 0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' & & \end{array} .$$

Show that there is an exact sequence

$$\ker a \longrightarrow \ker b \longrightarrow \ker c \longrightarrow \text{coker } a \longrightarrow \text{coker } b \longrightarrow \text{coker } c.$$