

Sheet 2

Mathematical Physics II

Exercise 1.

4 P.

Recall the WKB approximate solutions defined in the lecture

$$u_{\pm}(x, \hbar) := a_{\pm}(x)e^{i\phi_{\pm}(x)/\hbar}.$$

Construct a specific WKB solution by choosing $a_{\pm}^j(x)$ such that $u_{\pm}(x_0, \hbar) = 1$, where x_0 is the point for which $\exists U(x_0)$ such that

$$\phi_{\pm}(x) := \pm \int_{x_0}^x \sqrt{E - V(y)} \, dy$$

for all $x \in U(x_0)$.

Exercise 2.

4 P.

Recall the definition of the semi-classical Fourier transform of $u \in \mathcal{S}(\mathbb{R}^n)$, denoted by $\mathcal{F}_{\hbar}(u)$. Show that $\mathcal{F}_{\hbar}$ is a isomorphism of $\mathcal{S}'(\mathbb{R}^n)$.

Exercise 3.

4 P.

Show that $\mathcal{F}_{\hbar}$ is an isometry on $L^2(\mathbb{R}^n)$ and that

$$\mathcal{F}_{\hbar}(\hbar D_x u) = \xi \mathcal{F}_{\hbar} u, \quad \text{and} \quad \mathcal{F}_{\hbar}(xu) = -\hbar D_{\xi} \mathcal{F}_{\hbar} u.$$