Classical Solutions in Gauge Theories

The goal of this seminar is to explore both mathematical and physical aspects of gauge theories. The talks are aimed at a mixed audience of mathematics and physics students, with the aim of bridging the language gap between the two approaches to gauge theories.

To this end, the talks are labelled as "physics" or "math" talks. This emphasises the flavour of each talk, and is not meant to discourage mathematicians from giving physics talks or vice-versa.

Note that in the physics talks we will at least partly use the original literature and more details will be given during the Vorbesprechung.

1 The Yang-Mills equations for physicists

Talk 1: Maxwell's equations

Date: 21.10
Physics talk
Speaker: Jochum van der Bij

Talk 2: Lie groups and algebras

Date: 28.10
Physics talk: $SO(3)$ and $SU(2)$
Speaker: Jochum van der Bij

Talk 3: The Yang-Mills equations

Date: 4.11
Physics talk
Speaker: Jochum van der Bij

2 Mathematical formulation of the Yang-Mills equations

We follow Chapters 2 and 3 in Part II of [1] for a gentle introduction to the geometric concepts needed to reformulate the Yang-Mills equations in mathematical language.

Talk 4: Vector bundles

Date: 11.11
Math talk
Speaker: Lukas Hoffmann

- Introduce bundles, vector bundles, and G-bundles on manifolds and their sections

- Define dual bundles, direct sums, tensor products, and exterior products of bundles
- Discuss transition functions and cocycle conditions

References: Bundles and Vector Bundle Constructions from Part II, Chapter 2 of [1].

Talk 5: Symmetries of vector bundles and connections

Date: 18.11
Math talk
Speaker: Gregor Suchan

- Introduce the endomorphism bundle of a vector bundle and use it to define gauge transformations
- Define connections, G-connections, and gauge-equivalence of connections
- Write the vector potential A in terms of the endomorphism bundle

References: Gauge transformations and Connections from Part II, Chapter 2 of [1].

Talk 6: Holonomy

Date: 25.11
Math talk
Speaker: Mara Ungureanu

- Explain parallel transport along a path on a manifold
- Define the holonomy of a connection along a path
- Study the effect of gauge transformation on holonomy

References: Holonomy from Part II, Chapter 2 of [1]

Talk 7: Curvature

Date: 2.12
Math talk
Speaker: Mara Ungureanu

- Introduce the notion of curvature of a connection
- Write the curvature in terms of the vector potential
- Express the holonomy in terms of the curvature
- State (and if time permits prove) the Bianchi identity

References: Curvature and The Bianchi Identity from Part II, Chapter 3 of [1]

Talk 8: The Yang-Mills equations revisited

Date: 9.12
Math talk
Speaker: Lukas Hoffmann

- Write down the Yang-Mills equations using the language introduced in the previous talks
- Recover Maxwell's equations as a special case

References: The Yang-Mills Equations from Part II, Chapter 3 of [1]

3 Symmetry breaking

Talk 9: Skyrmeons

Date: 16.12
Physics talk
Speaker: Yann Stoll

Talk 10: The Aharonov-Bohm effect and the Dirac monopole

Date: 13.01
Physics talk
Speaker: Jochum van der Bij

Talk 11: The Nielsen-Olesen string

Date: 20.01
Physics talk
Speaker: Tobias Lippold

Talk 12: The 't Hooft-Polyakov monopole

Date: 27.01
Physics talk
Speaker: Zainab Chokr

Talk 13: Higgs fields and symmetry breaking

Date: 3.02
Math talk
Speaker: Mara Ungureanu

- Describe the symmetry breaking mechanism geometrically
- This means: explain G-bundle reduction, rewrite the Lagrangian in terms of quantities defined on the reduced bundle, and finally interpret some of the new terms that appear as massive fields, thus completing the description of the mass generating (Higgs) mechanism

References: Chapter V Bis, Section 5 from [2]

4 Instantons

Talk 14: Instanton solutions

Date: 10.02
Physics talk
Speaker: Jochum van der Bij

References

[1] J. Baez, J.P. Muniain. *Gauge Fields, Knots and Gravity,* World Scientific, 1994.

[2] Y. Choquet-Bruhat, C. DeWitt-Morrette. *Analysis, Manifolds and Physics, Part II,* Elsevier, 2000.

[3] C. Nash, S. Sen. *Topology and Geometry for Physicists,* Academic Press, 1983.