Exercise Sheet 10 (bonus)
19. Juli 2024

Due on August 5 in my mailbox or via email. Each problem is worth two points.

Exercise 1 (Rosenstein 9.16)**.** If L is a linear order, then we say that L is a *fixed point* if L has an element a such that $f(a) = a$ for every order-preserving map $f : L \to L$. Show that if L is a fixed point, then L is not order-isomorphic to $L + L$.

Exercise 2. Show that if $\theta > 2^{\aleph_0}$ is a regular cardinal and $\mathbb{R} \in M \prec H(\theta)$, then $M \cap (\mathbb{R} \setminus \mathbb{Q})$ is dense in $\mathbb{R}$.

Exercise 3 (Old Kunen II.25)**.** Assume MA. Show that if $\mathcal{A}$ is a family of Lebesgue-measurable sets with $|\mathcal{A}| < 2^{\aleph_0}$, then $\bigcup \mathcal{A}$ is Lebesgue-measurable and there is some countable $\mathcal{B} \subseteq \mathcal{A}$ such that $\mu(\bigcup \mathcal{A}) = \mu(\bigcup \mathcal{B})$.

Definition 1. We say that $\diamondsuit$ holds if there is a sequence $\langle X_\alpha : \alpha < \omega_1 \rangle$ such that for all $Y \subseteq \omega_1$, the set $\{\alpha < \omega_1 : Y \cap \alpha = X_\alpha\}$ is stationary.

Exercise 4. Prove that $\diamondsuit$ implies $2^{\aleph_0} = \aleph_1$.