

Exercise Sheet 3

May 7, 2024

Due on May 13 *before* the exercise session.

Note: The previous version was missing a detail in Exercise 1 that rendered the problem quite trivial.

Exercise 1 (2 points). Suppose that $P \subseteq \mathbb{R}$ is perfect, meaning that it is closed and has no isolated points. Show that if $P \cap (a, b) \neq \emptyset$ for some open interval $(a, b) \subseteq \mathbb{R}$, then $|P \cap (a, b)| = 2^{\aleph_0}$. (Hint: You can use the Cantor-Bendixson Theorem as part of your argument, but you do not need to repeat the whole construction.)

Exercise 2 (2 points). Show that there are at least 2^{\aleph_0} -many distinct countable linear orders up to isomorphism.