Exercise Sheet 5
27. Mai 2024

Due on June 3 before the exercise session.

Exercise 1 (2 points)**.** Show that Ramsey's Theorem does not generalize to finite subsets of $\mathbb{N}$ as follows: Show that there is a function f with domain $\mathbb{N}^{<\mathbb{N}}$ (i.e. finite subsets of natural numbers) such that there is no infinite $X \subseteq \mathbb{N}$ that is homogeneous for f (i.e. if $X \subseteq \mathbb{N}$ is infinite, then it is not the case that for all finite $s, t \subseteq X$, $f(s) = f(t)$).

(Hint/giveaway: Let $f(s) = 1$ if $|s| \in s$.)

Exercise 2. Show that $\mathbb{R}^+$ (i.e. the positive real numbers) can be decomposed into two disjoint sets of size $2^{\aleph_0}$ that are both closed under addition. In other words, $\mathbb{R}^+ = A \cup B$ where $|A| = |B| = 2^{\aleph_0}$, $A \cap B = \emptyset$, for all $x, y \in A$ we have $x + y \in A$, and similarly we have such a closure for B.