Exercise Sheet 6 3. Juni 2024

Due on June 10 before the exercise session.

Exercise 1 (2 points). Suppose θ is an infinite cardinal. If $x \in H(\theta)$ and $2^{|x|} < \theta$ then for all $y \subseteq x, y \in H(\theta)$.

Exercise 2 (3 points). Assume θ is an infinite cardinal and prove the following:

- 1. $H(\theta)$ is transitive, meaning that if $b \in a \in H(\theta)$ then $b \in H(\theta)$,
- 2. $H(\theta)$ is a set (i.e. $H(\theta)$ can be explicitly constructed),
- 3. $H(\theta) \cap ON = \theta$ (where ON stands for the class of ordinals).

Exercise 3 (2 points). Suppose θ is an infinite regular cardinal. If $F : H(\theta) \to H(\theta)$ is any function, then for any set $X \in H(\theta)$, $F[X] := \{F(z) : z \in X\} \in H(\theta)$. (Hint: Use the fact that θ is regular.)