Exercise Sheet 9
4. Juli 2024

Due on July 8 before the exercise session.

Exercise 1 (Jech 16.16, 2 points per item)**.** Assume $\mathsf{MA} \wedge 2^{\aleph_0} = \aleph_2$ and let $\{X_\alpha : \alpha < \aleph_1\}$ be a sequence of infinite subsets of ω such that $X_\beta \setminus X_\alpha$ is finite if $\alpha < \beta$. Show that there exists an infinite X such that $X \setminus X_\alpha$ is finite for all $\alpha < \aleph_1$.

This uses the following poset $\mathbb{P}$: The elements of the poset are pairs (s, F) where s is a finite subset of ω and F is a finite subset of $\aleph_1$. We have $(s', F') \leq (s, F)$ if $s' \supseteq s$, $F' \supseteq F$, and $s' \setminus s \subseteq X_\alpha$ for all $\alpha \in F$.

1. Show that $\mathbb{P}$ has the countable chain condition.
2. Show that the relevant sets are dense (they are in Jech if you want to look) and show that the application of Martin's Axiom gives the infinite set X.