Machine Learning and Mathematical Logic
Winter term 2025-2026 Lecturer: Maxwell Levine
Exercises

Exercise Sheet 1

Due at the beginning of the exercise session on 22.10.2025 at 4:20. Any problem is worth 4 points unless specified otherwise. A total of at least 50% of all available points is required for the "Studienleistung" (we expect there to be 12 or 13 sheets).

Here are some statements that we will want for a proof in the lecture.

You can assume the random variables below have finite range.

Exercise 1 (1 point)**.** Prove Markov's inequality: If X is a non-negative random variable, then for all $a \geq 0$,

$$\mathbb{P}[X \geq a] \leq \frac{\mathbb{E}[X]}{a}$$

where $\mathbb{E}$ is the expected value and $\mathbb{P}[X \geq a]$ is the probability that X will output a value greater or equal to a.

Exercise 2 (2 points)**.** Let X be a random variable that takes values in $[0,1]$ and let $\mu = \mathbb{E}[X]$. Then for any $a \in (0,1)$, then

$$\mathbb{P}[X > 1 - a] \geq \frac{\mu - (1-a)}{a}$$

and

$$\mathbb{P}[X > a] \geq \frac{\mu - a}{1 - a} \geq \mu - a$$

where the second inequality holds *if* $\mu > a$.

Exercise 3 (2 points)**.** Let X be a random variable that takes values in $[0,1]$ and whose expected value satisfies $\mathbb{E}[X] \geq 1/4$. Then $\mathbb{P}[X \geq 1/8] \geq 1/7$.