Machine Learning and Mathematical Logic
Winter term 2025-2026 Lecturer: Maxwell Levine
Exercises

Exercise Sheet 12

Due at the beginning of the exercise session at 16:20. A total of at least 50% of all available points is required for the "Studienleistung" (I expect there to be 12 or 13 sheets).

Exercise 1 (Kechris, Exercise 3.2, Proposition 3.3, 1 point each)**.**

1. Consider the open interval $(0, 1)$ with its usual topology. Show that it is Polish although its usual metric is not complete. (Meaning that we consider the topology generated by open intervals (a, b) for $0 < a < b < 1$, but the metric $d(a, b) := |b - a|$ does not prove that this is a Polish space.)

2. Show that a closed subspace of a Polish space is a Polish space.

Exercise 2 (2 points)**.** Show that if X is a Polish space and $\alpha < \beta$, then the following containments hold: $\mathbf{\Sigma}^0_\alpha(X) \subseteq \mathbf{\Sigma}^0_\beta(X)$, $\mathbf{\Sigma}^0_\alpha(X) \subseteq \mathbf{\Pi}^0_\beta(X)$, $\mathbf{\Pi}^0_\alpha(X) \subseteq \mathbf{\Pi}^0_\beta(X)$, and $\mathbf{\Pi}^0_\alpha(X) \subseteq \mathbf{\Sigma}^0_\beta(X)$.

Exercise 3 (Jech 11.5, 2 bonus points)**.** Show that if A has the Baire property, then there exist sets G and F such that $G \subseteq A \subseteq F$, G is a countable intersection of open sets, F is a countable union of closed sets, and the difference $F \setminus G$ is meager.