Machine Learning and Mathematical Logic
Winter term 2025-2026 Lecturer: Maxwell Levine
Exercises

Exercise Sheet 2

Due at the beginning of the exercise session on 22.10.2025 at 16:20. Any problem is worth 4 points unless specified otherwise. A total of at least 50% of all available points is required for the "Studienleistung" (I expect there to be 12 or 13 sheets).

The first exercise is in Ben-David and Shalev-Shwartz as Lemmas A.1 and A.2, but if you look at their proofs, please add details.

Exercise 1 (2 points)**.**

1. Let $a > 0$. Then $x \geq 2a\log(a) \implies x \geq a\log(x)$. (Hint: Consider the claim first for $0 < a \leq \sqrt{e}$. For $a > \sqrt{e}$ consider $f(x) = x - alog(x)$. Consider $f'(x)$ and $f(2a\log(a))$.)
2. Let $a \geq 1$ and $b > 0$. Then $x \geq 4a\log(2a) + 2b \implies x \geq a\log(x) + b$. (Hint: It suffices to prove that $x \geq 4a\log(2a) + 2b$ implies that both $x \geq 2a\log(x)$ and $x \geq 2b$ and use the first point of the exercise.)

Exercise 2 (BDSS Ex. 3.1, 2 points)**.** Let $\mathcal{H} \subseteq \mathcal{X} \times \{0,1\}$ be a binary classification class. Assume that $\mathcal{H}$ is PAC learnable with its sample complexity given by $m_{\mathcal{H}}(\cdot,\cdot)$. Show that $m_{\mathcal{H}}$ is monotonically *non*increasing in both of its arguments, i.e.

1. given $\delta \in (0,1)$ and $\epsilon_1, \epsilon_2 \in (0,1)$ with $\epsilon_1 \leq \epsilon_2$, show that $m_{\mathcal{H}}(\epsilon_1, \delta) \geq m_{\mathcal{H}}(\epsilon_2, \delta)$, and
2. given $\epsilon \in (0,1)$ and $\delta_1, \delta_2 \in (0,1)$ with $\delta_1 \leq \delta_2$, we have $m_{\mathcal{H}}(\epsilon, \delta_1) \geq m_{\mathcal{H}}(\epsilon, \delta_2)$.

Exercise 3 (MRT Ex. 2.3, 3 points)**.** Consider the problem of learning concentric circles. Let $\mathcal{X} = \mathbb{R}^2$ and consider concepts of the form $\{(x,y) : x^2 + y^2 \leq r^2\}$ for some real number. Show that this class is PAC learnable with sample complexity given by $m \geq (1/\epsilon)\log(1/\delta)$.