Exercise Sheet 5

Due at the beginning of the exercise session at 16:20. A total of at least 50% of all available points is required for the "Studienleistung" (I expect there to be 12 or 13 sheets).

Exercise 1 (Marker 1.4.10, 1 point each)**.** Let $\mathbb{M}$ be an $\mathcal{L}$-structure and $A \subseteq M$. We say that $b \in M$ is *definable* over A if there is a formula $\phi(v, \bar{w})$ and $\bar{a} \in A$ such that

$$\mathbb{M} \models \phi(b, \bar{a}) \wedge \forall y(\phi(y, \bar{a}) \rightarrow y = b).$$

In other words, $\{b\}$ is A-definable.

(a) Show that x is definable over A if and only if there is a 0-definable (i.e. definable without parameters) X, a 0-definable $f : X^n \rightarrow M$, and an n-tuple $\bar{a} \in A$ with $f(\bar{a}) = x$.

(b) Suppose that x is definable from A and σ is an automorphism of M (i.e. an isomorphism from M to itself) such that $\sigma(a) = a$ for all $a \in A$. Show that $\sigma(x) = x$.

(c) Let $\operatorname{dcl}(A) = \{x \in M \mid x \text{ is definable from } A\}$. Show that $\operatorname{dcl}(\operatorname{dcl}(A)) = \operatorname{dcl}(A)$.

Exercise 2 (Marker 2.5.2, 2 points)**.** Suppose that T has arbitrarily large finite models. Show that T has an infinite model.