Exercise Sheet 6

Due at the beginning of the exercise session at 16:20. A total of at least 50% of all available points is required for the "Studienleistung" (I expect there to be 12 or 13 sheets).

Exercise 1 (Marker 3.4.4, 2 points)**.** Consider the natural numbers with the function $s(x) = x+1$. Show that the theory of $(\mathbb{N}, s)$ does not have quantifier elimination.

Exercise 2 (2 points)**.** Show that if $\mathcal{L} = \{R\}$ and T is the theory of the random graph, then the formula $\varphi(x, y) = xRy$ is IP.

Exercise 3 (1 point)**.** Show that Ramsey's Theorem does not generalize to finite subsets of $\mathbb{N}$ as follows: Show that there is a function f with domain $\mathbb{N}^{<\mathbb{N}}$ (i.e. finite subsets of natural numbers) such that there is no infinite $X \subseteq \mathbb{N}$ that is homogeneous for f (i.e. if $X \subseteq \mathbb{N}$ is infinite, then it is not the case that for all finite $s, t \subseteq X$, $f(s) = f(t)$).

(Hint/giveaway: Let $f(s) = 1$ if $|s| \in s$.)